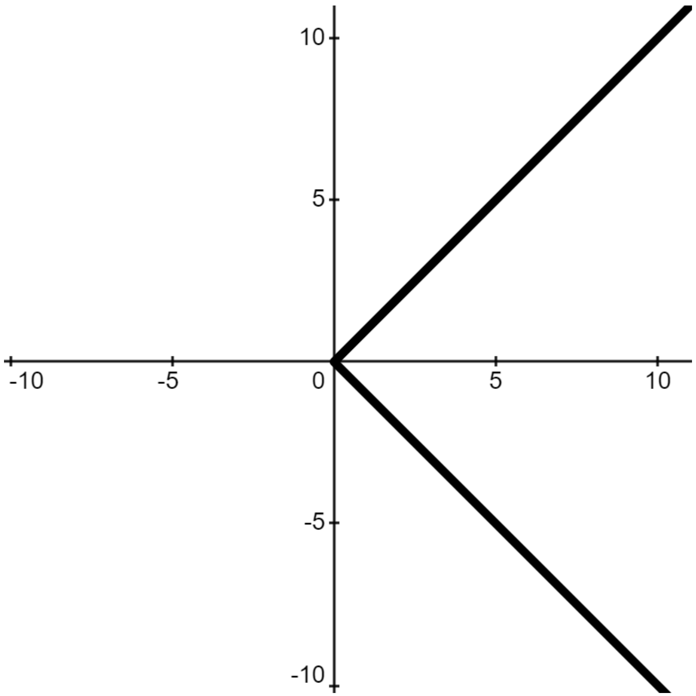
Friedrich-Alexander-Universität
Erlangen-Nürnberg

Converse Unruh effect: Wedge-modular inclusions and thermal states

DPG-Frühjahrstagung Berlin

Ian Koot

Unruh Effect in (Free) QFT: set up



Suppose we have some Free (Bosonic) field Theory on $\mathbb{R}^2$: $\mathcal{H}_1$ is the one-particle space, $\mathcal{H} := \mathcal{F}_+(\mathcal{H}_1)$, and

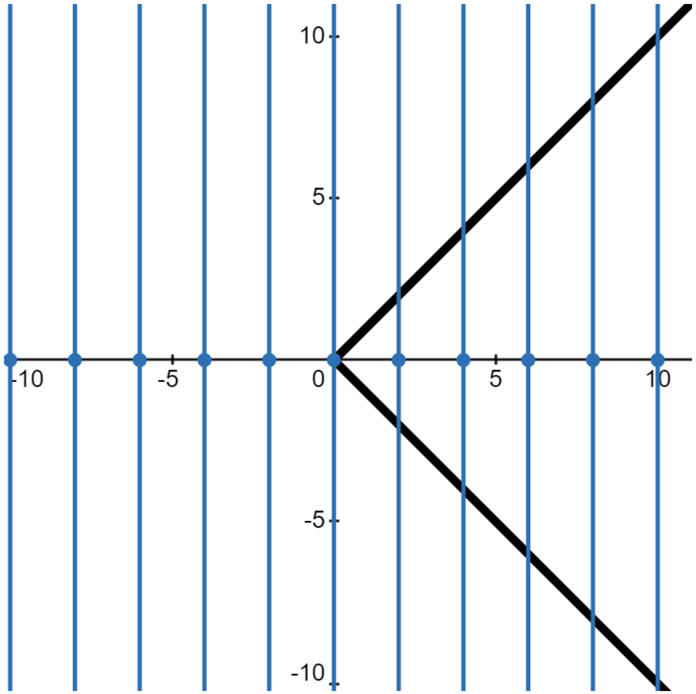
$$\mathcal{A} := \{\exp(i\phi(f)) : f \in C_c^\infty(\mathbb{R}^2)\} \subset B(\mathcal{H}).$$

Additionally, we consider localized fields

$$\mathcal{A}(W) := \{\exp(i\phi(f)) : f \in C_c^\infty(W)\} \subset \mathcal{A}$$

We consider the free time evolution $\alpha_t(\phi(\vec{x})) = \phi(\vec{x} + (t, 0))$, or $\alpha_t(X) = e^{itH} X e^{-itH}$. The vacuum state $\Omega \in \mathcal{H}$ is a ground state for H.

Unruh Effect in (Free) QFT: set up



Suppose we have some Free (Bosonic) field Theory on $\mathbb{R}^2$: $\mathcal{H}_1$ is the one-particle space, $\mathcal{H} := \mathcal{F}_+(\mathcal{H}_1)$, and

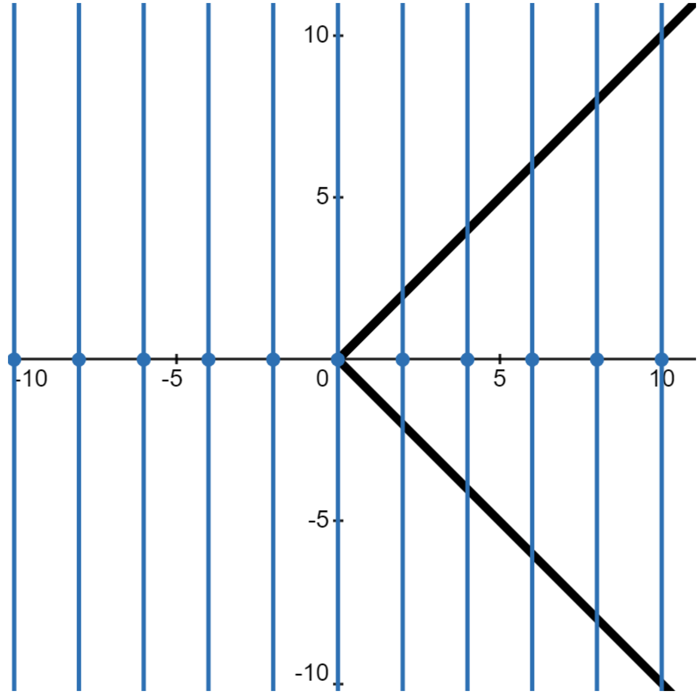
$$\mathcal{A} := \{\exp(i\phi(f)) : f \in C_c^\infty(\mathbb{R}^2)\} \subset B(\mathcal{H}).$$

Additionally, we consider localized fields

$$\mathcal{A}(W) := \{\exp(i\phi(f)) : f \in C_c^\infty(W)\} \subset \mathcal{A}$$

We consider the free time evolution $\alpha_t(\phi(\vec{x})) = \phi(\vec{x} + (t, 0))$, or $\alpha_t(X) = e^{itH} X e^{-itH}$.
The vacuum state $\Omega \in \mathcal{H}$ is a ground state for H.

Unruh Effect in (Free) QFT: thermal state

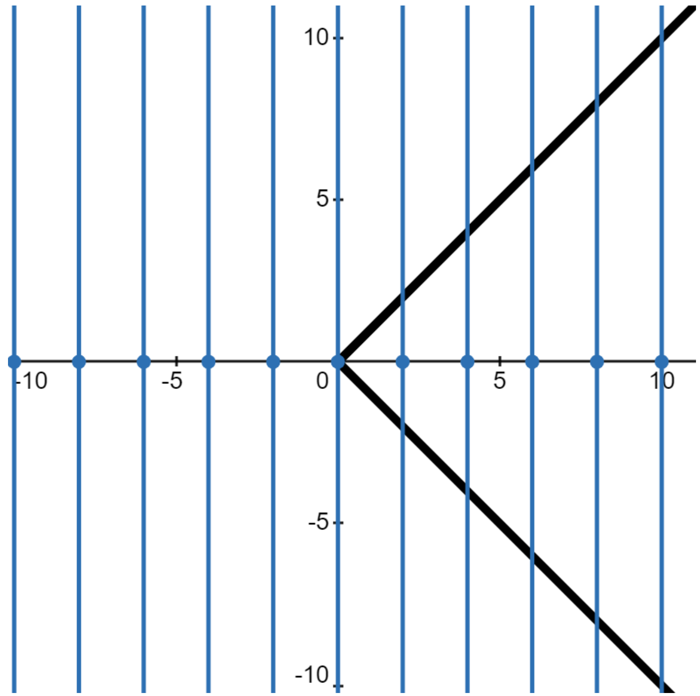


- A KMS/thermal state ω_β at inverse temperature β with respect to a time evolution σ_t means: for $A, B \in \mathcal{A}$, we have

  $$\omega_\beta(A\sigma_t(B)) = \omega_\beta(\sigma_{t+i\beta}(B)A).$$

  This is the proper generalization of Gibbs states.

Unruh Effect in (Free) QFT: thermal state

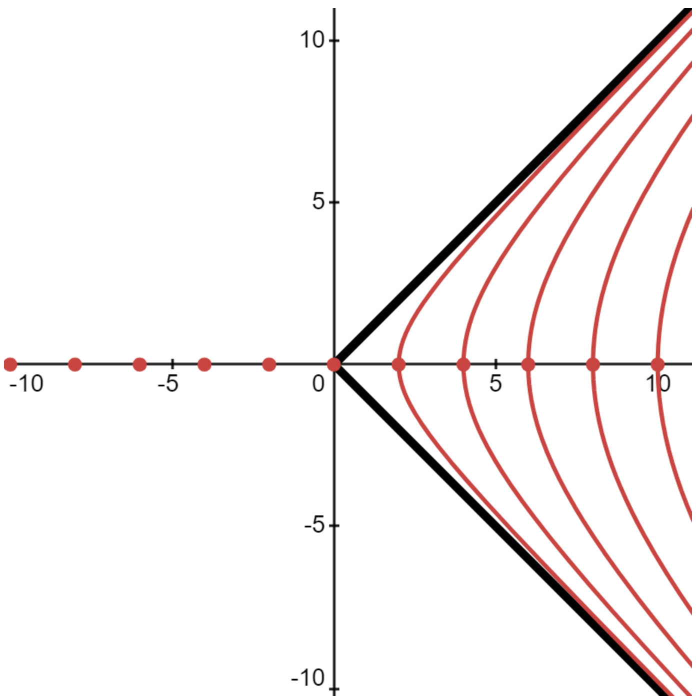


- A KMS/thermal state ω_β at inverse temperature β with respect to a time evolution σ_t means: for $A, B \in \mathcal{A}$, we have

$$\omega_\beta(A\sigma_t(B)) = \omega_\beta(\sigma_{t+i\beta}(B)A).$$

  This is the proper generalization of Gibbs states.
- The Unruh effect can be formulated as follows (Sewell 1982): on $\mathcal{A}(W)$, the state $A \mapsto \langle \Omega, A\Omega \rangle$ agrees with a **thermal state** ω_β which is in equilibrium with respect to a **different** time evolution σ_t.

Unruh Effect in (Free) QFT: thermal state



- A KMS/thermal state ω_β at inverse temperature β with respect to a time evolution σ_t means: for $A, B \in \mathcal{A}$, we have

$$\omega_\beta(A\sigma_t(B)) = \omega_\beta(\sigma_{t+i\beta}(B)A).$$

  This is the proper generalization of Gibbs states.
- The Unruh effect can be formulated as follows (Sewell 1982): on $\mathcal{A}(W)$, the state $A \mapsto \langle \Omega, A\Omega \rangle$ agrees with a **thermal state** ω_β which is in equilibrium with respect to a **different** time evolution σ_t.
- In this specific case, this new time evolution is given by $\sigma_t(\phi(\vec{x})) = \phi(\Lambda_t \vec{x})$ (the Lorentz boost).

More general: Modular Theory

- Suppose $\mathcal{A} \subset B(\mathcal{H})$, and let $\Omega \in \mathcal{H}$ be cyclic and separating ($\overline{\mathcal{A}\Omega} = \mathcal{H}$ and $A\Omega = 0 \Rightarrow A = 0$).

More general: Modular Theory

- Suppose $\mathcal{A} \subset B(\mathcal{H})$, and let $\Omega \in \mathcal{H}$ be cyclic and separating ($\overline{\mathcal{A}\Omega} = \mathcal{H}$ and $A\Omega = 0 \Rightarrow A = 0$).
- We define an (unbounded) anti-linear operator

$$S : \mathcal{A}\Omega \to \mathcal{A}\Omega : A\Omega \mapsto A^*\Omega.$$

and we set $\Delta = S^*S$.

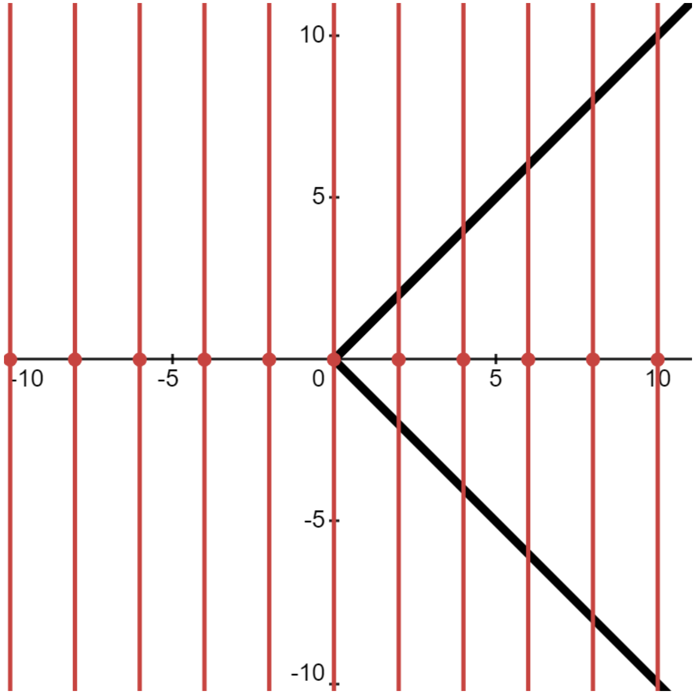
More general: Modular Theory

- Suppose $\mathcal{A} \subset B(\mathcal{H})$, and let $\Omega \in \mathcal{H}$ be cyclic and separating ($\overline{\mathcal{A}\Omega} = \mathcal{H}$ and $A\Omega = 0 \Rightarrow A = 0$).
- We define an (unbounded) anti-linear operator

$$S : \mathcal{A}\Omega \to \mathcal{A}\Omega : A\Omega \mapsto A^*\Omega.$$

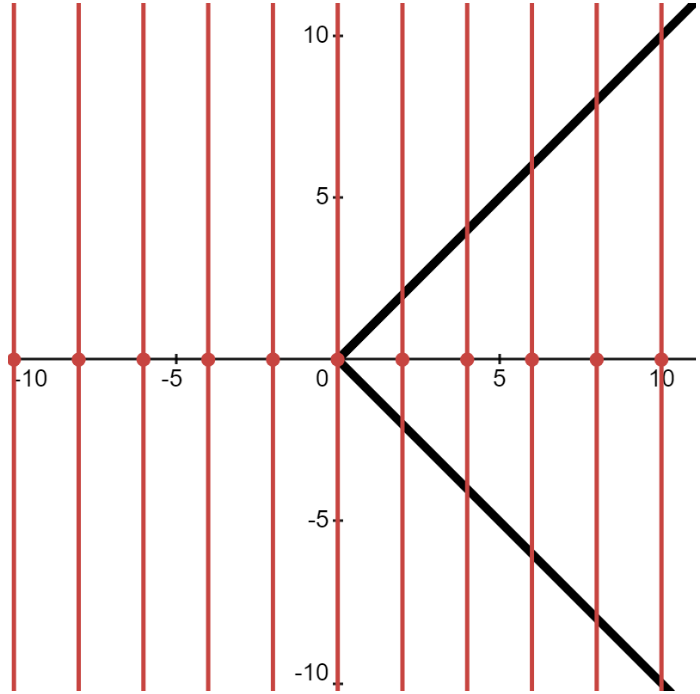
  and we set $\Delta = S^*S$.
- Let $\sigma_t(A) := \Delta^{it} A \Delta^{-it}$. It turns out that $\sigma_t(\mathcal{A}) = \mathcal{A}$, so this defines a 'time evolution'. It turns out: $A \mapsto \langle \Omega, A\Omega \rangle$ is KMS for σ_t! (but in general it won't be clear what σ_t actually looks like)

Converse



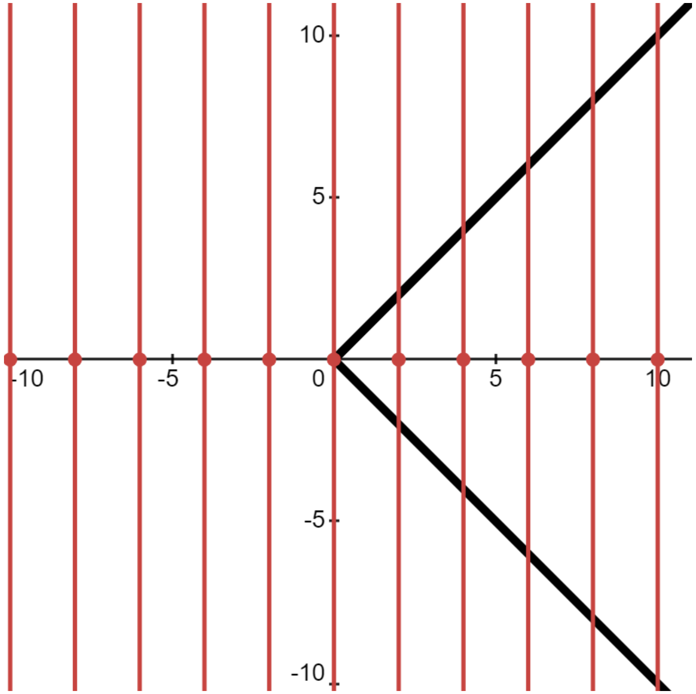
- Now suppose we have a thermal vector Ω_β for $\mathcal{A}$ with respect to a time evolution $\sigma_t(A) = e^{itH} A e^{-itH}$. Can we (formally) do the converse: find a time evolution α_t such that Ω_β is a ground state?

Converse



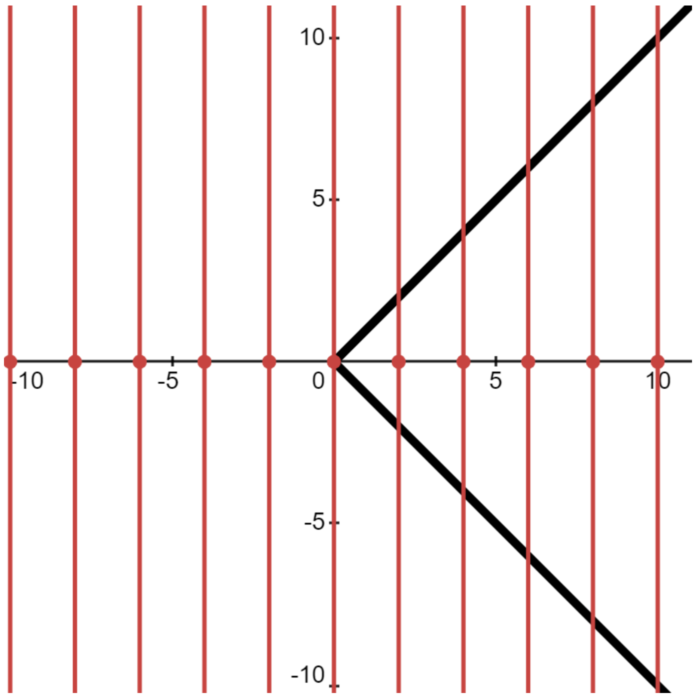
- Now suppose we have a thermal vector Ω_β for $\mathcal{A}$ with respect to a time evolution $\sigma_t(A) = e^{itH} A e^{-itH}$. Can we (formally) do the converse: find a time evolution α_t such that Ω_β is a ground state?
- As we saw before, probably cannot preserve the algebra $\mathcal{A}$, it will be “moving off to infinity” in finite time. Also, the trivial representation does the job; but with a wedge-like subalgebra we can construct a nontrivial representation.

Converse



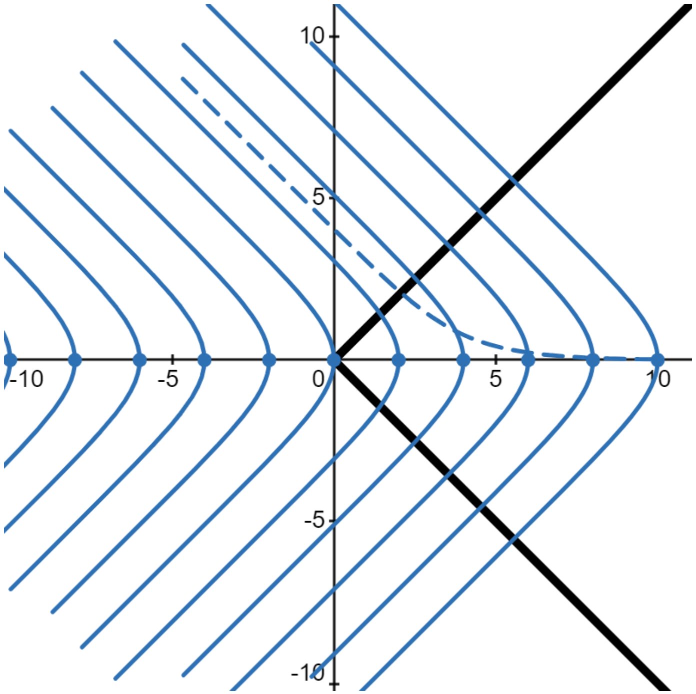
- Now suppose we have a thermal vector Ω_β for $\mathcal{A}$ with respect to a time evolution $\sigma_t(A) = e^{itH} A e^{-itH}$. Can we (formally) do the converse: find a time evolution α_t such that Ω_β is a ground state?
- As we saw before, probably cannot preserve the algebra $\mathcal{A}$, it will be "moving off to infinity" in finite time. Also, the trivial representation does the job; but with a wedge-like subalgebra we can construct a nontrivial representation.
- Original question: what is the modular time evolution in $\mathcal{A}(W)$?

Example: Free Chiral Theory



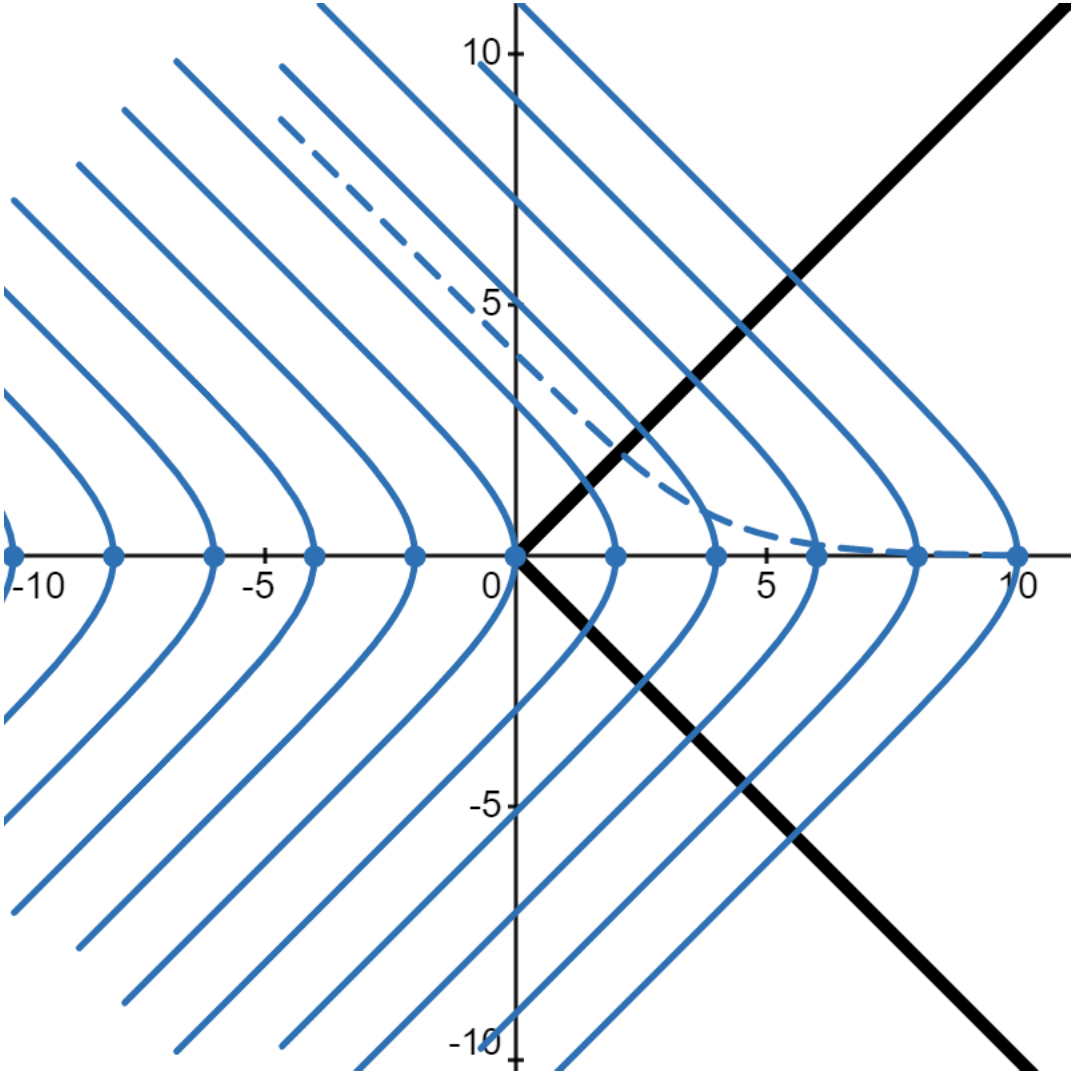
- An example calculated by Borchers and Yngvason (1999): a 2-dimensional free chiral theory, so $\mathcal{A} = \mathcal{A}_L \otimes \mathcal{A}_R$ where $\mathcal{A}_L$ and $\mathcal{A}_R$ consist of fields existing on the Left and Right light ray.

Example: Free Chiral Theory

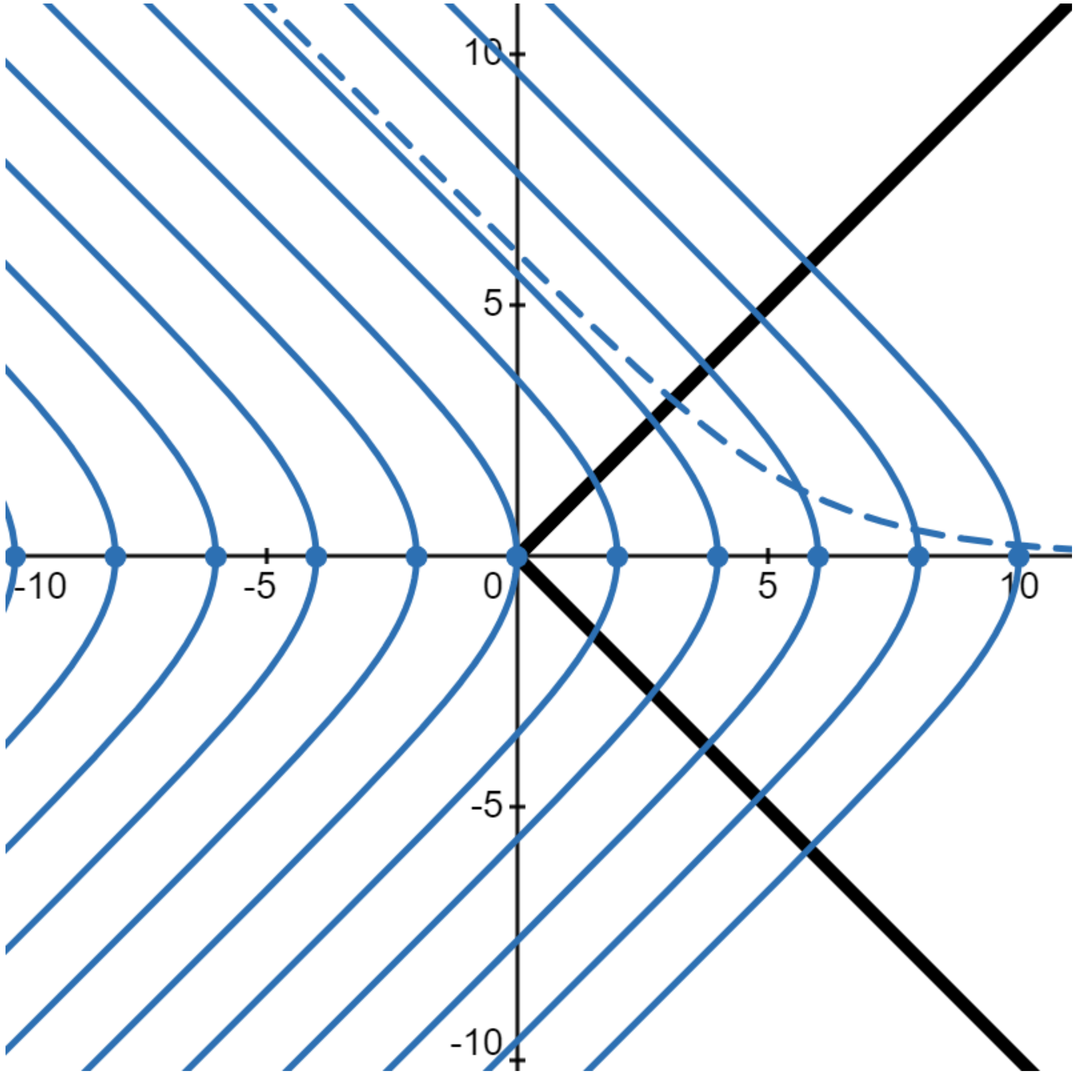


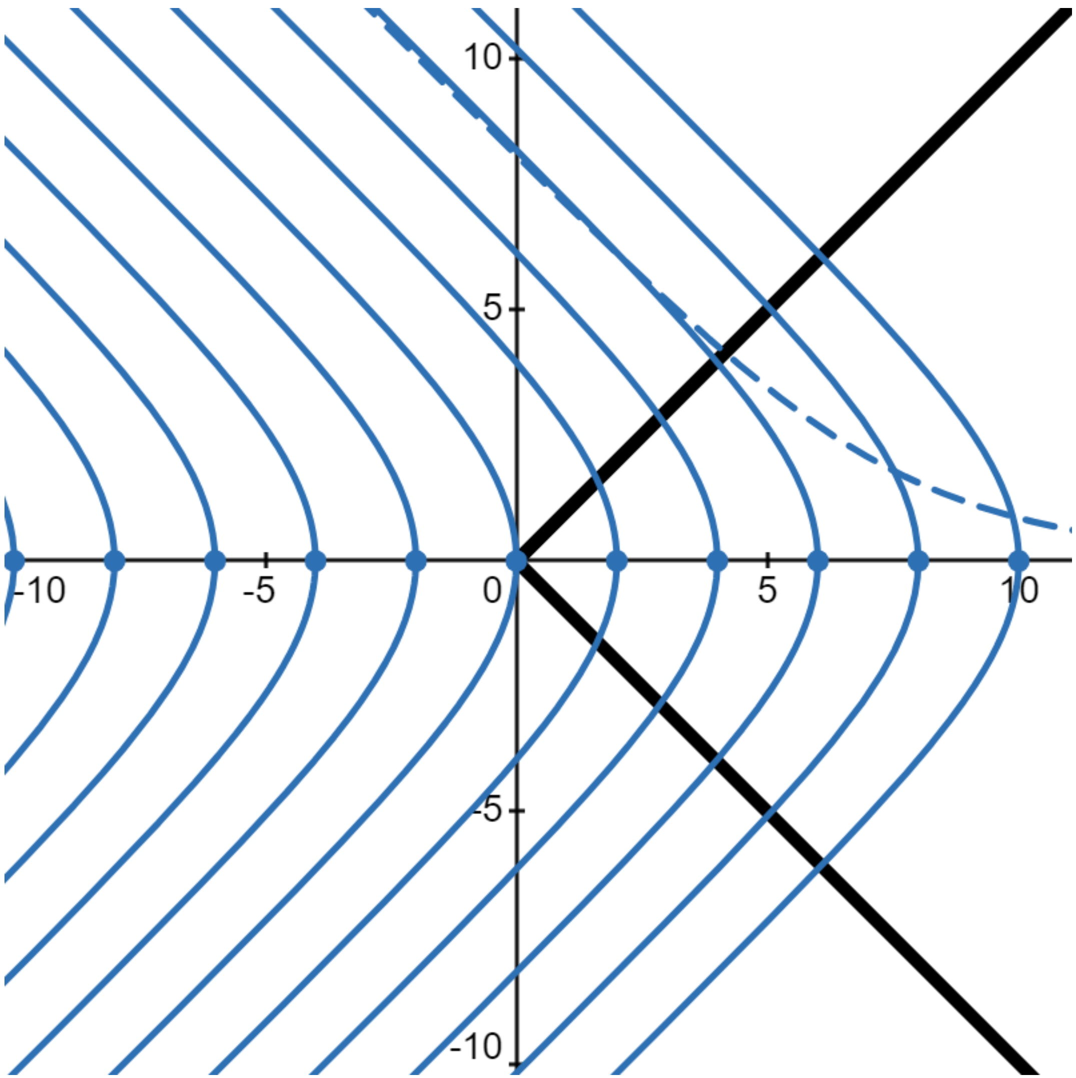
- An example calculated by Borchers and Yngvason (1999): a 2-dimensional free chiral theory, so $\mathcal{A} = \mathcal{A}_L \otimes \mathcal{A}_R$ where $\mathcal{A}_L$ and $\mathcal{A}_R$ consist of fields existing on the Left and Right light ray.
- If we consider the thermal state at temperature β then the evolution we get is given by

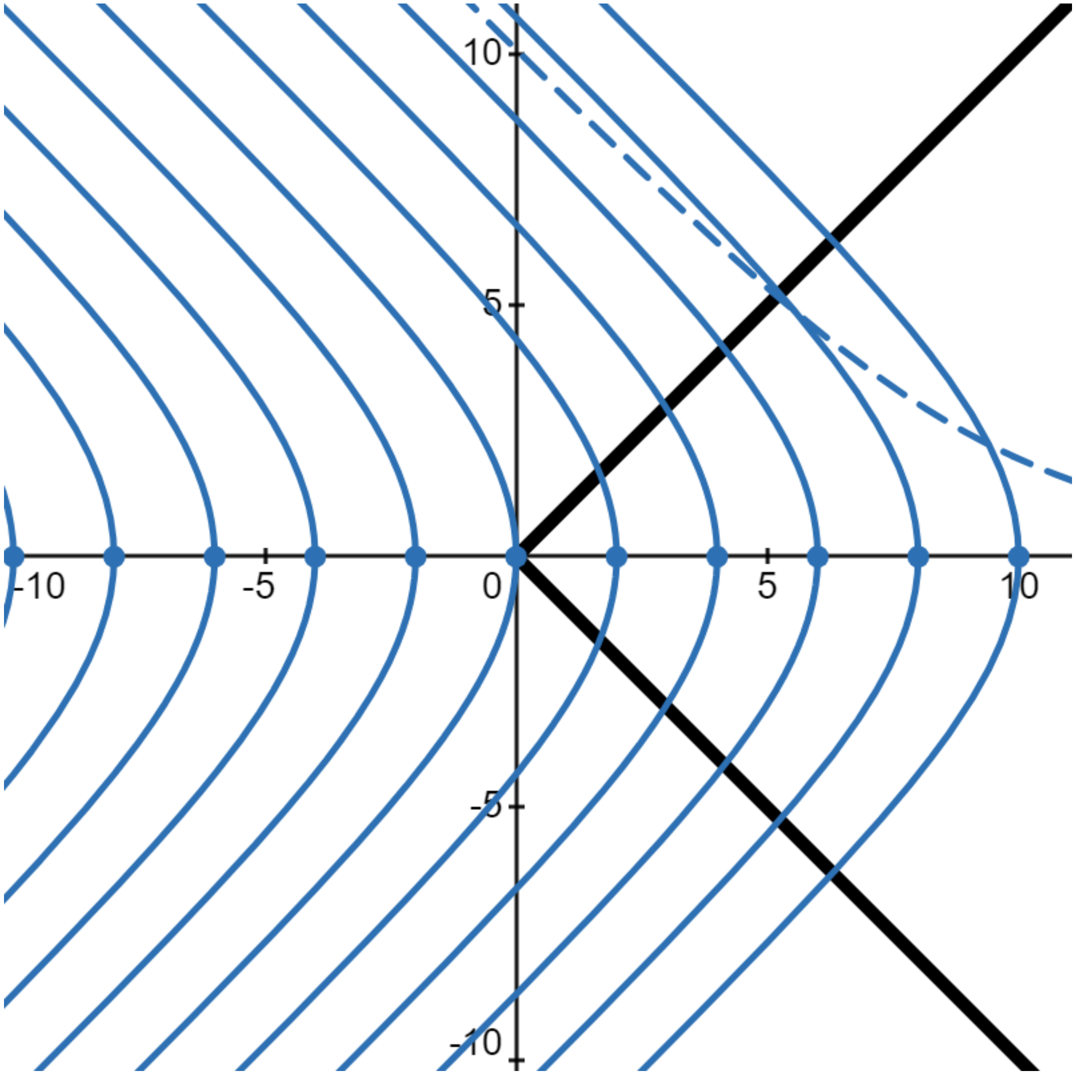
$$\gamma_t^{(R)}(x_R) = \frac{\beta}{2\pi} \ln\left(e^{2\pi x_R/\beta} - t\right)$$
$$\gamma_t^{(L)}(x_L) = -\frac{\beta}{2\pi} \ln\left(e^{-2\pi x_L/\beta} + t\right)$$

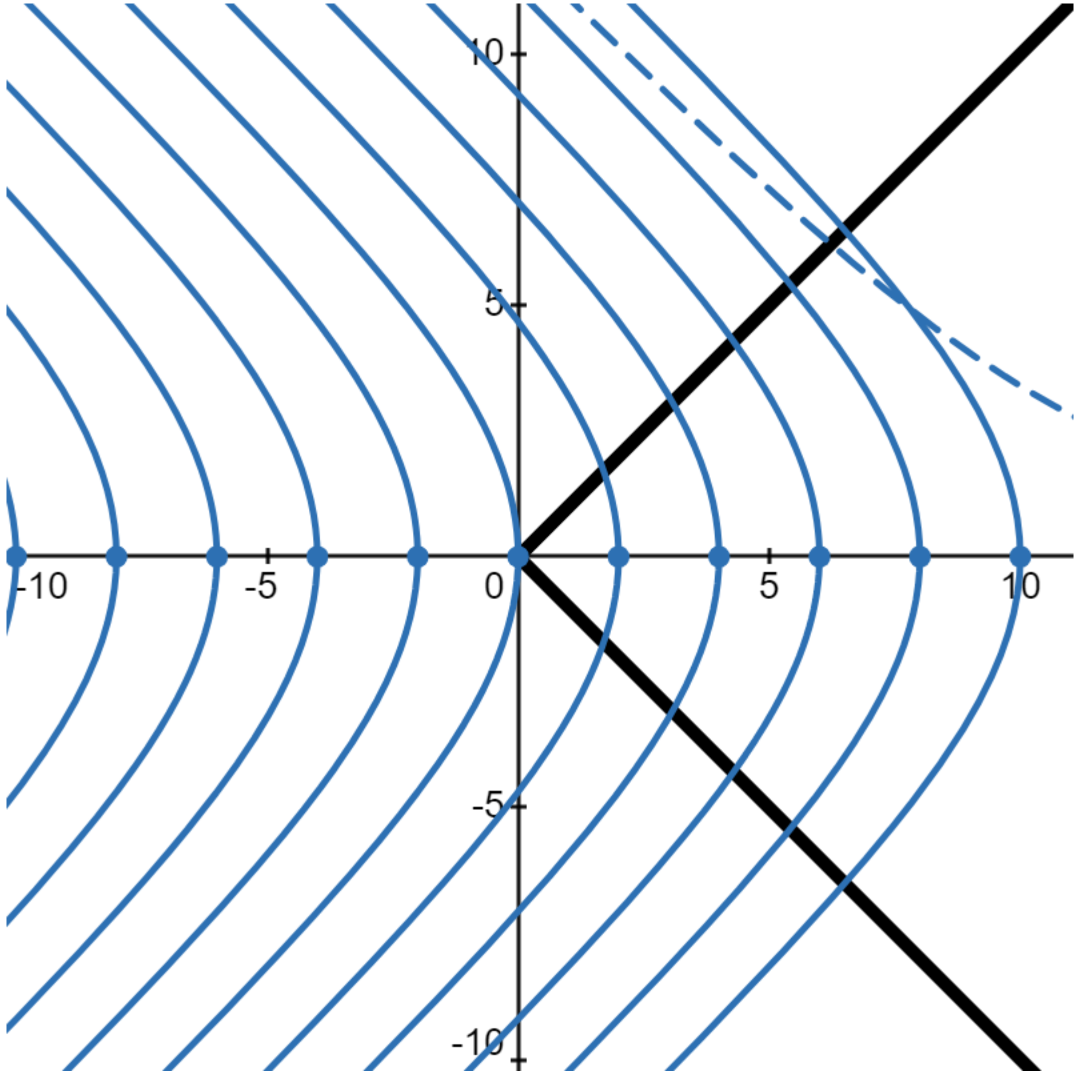


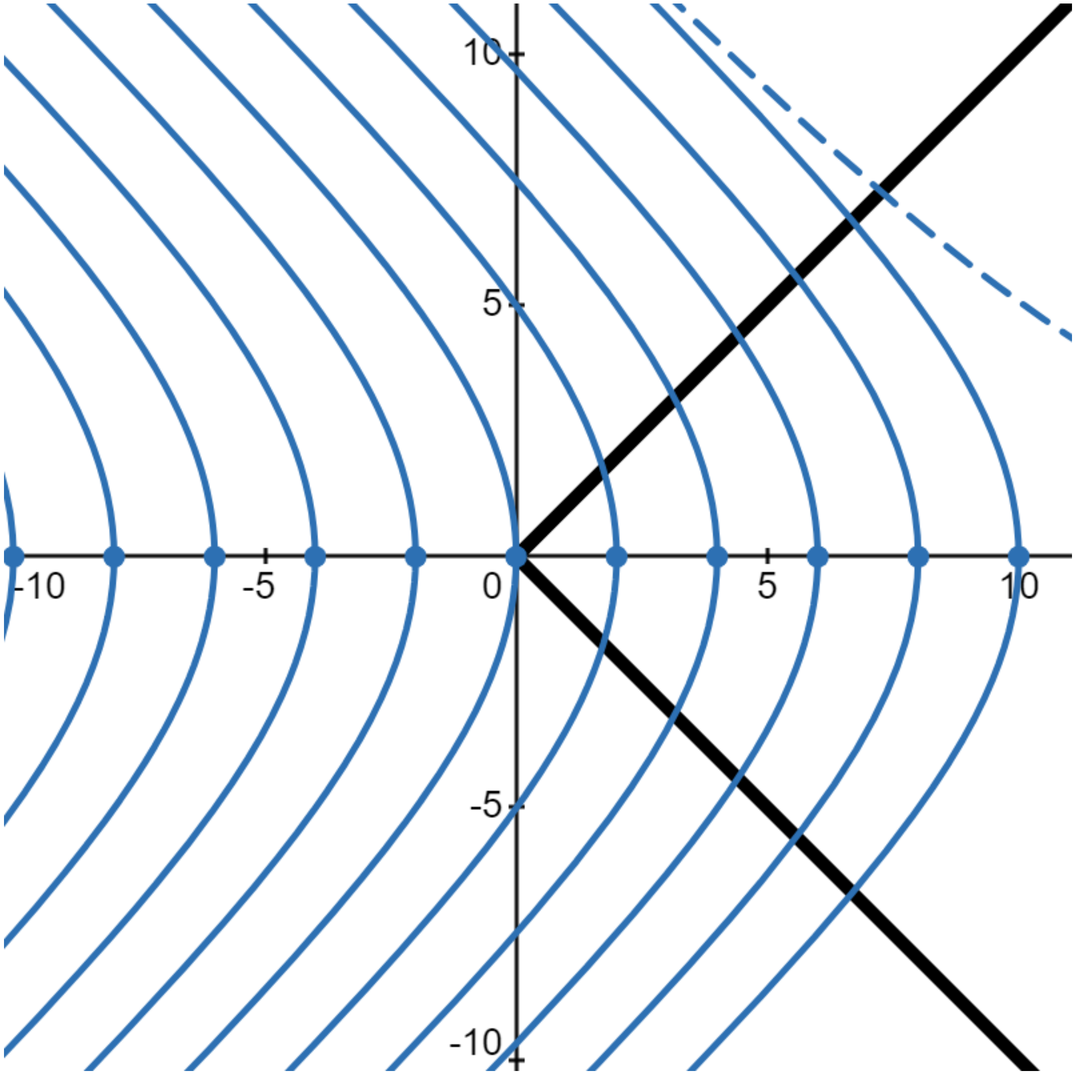
10
5
-5
-10
-10
-5
0
5
10

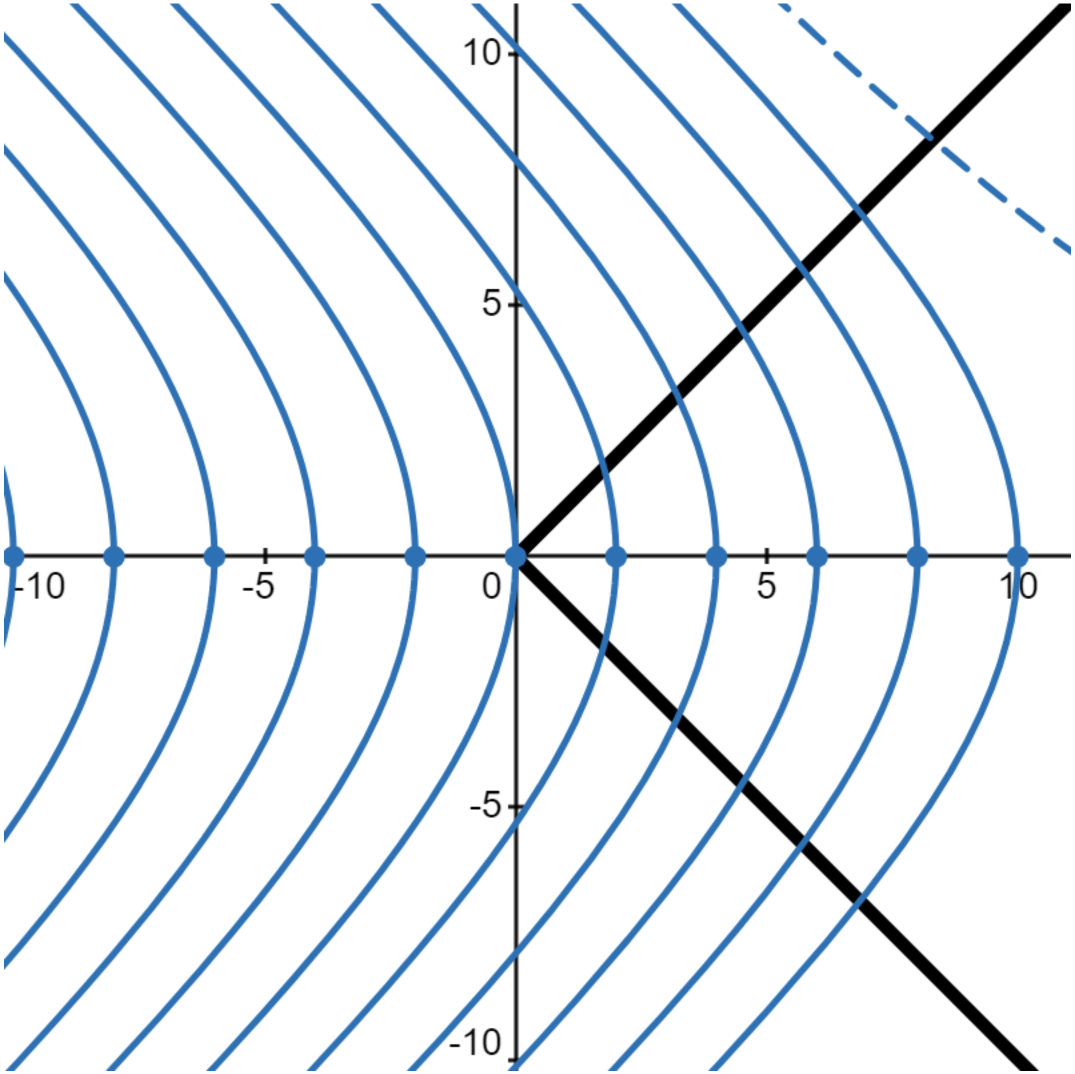

10
5
-5
-10
-10
-5
0
5
10


10
5
-5
-10
-10
-5
0
5
10

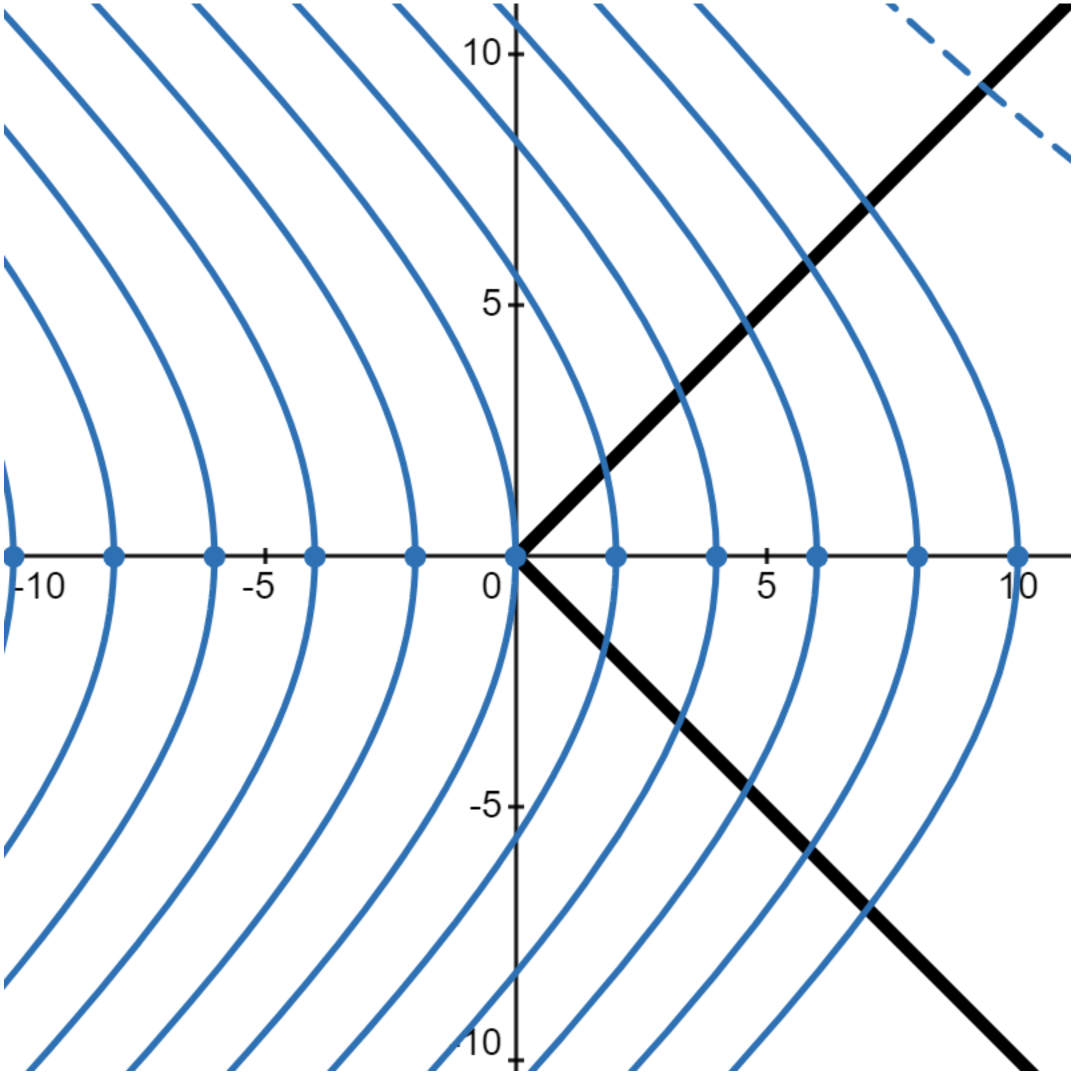

10
5
-5
-10
-10
-5
0
5
10


10
5
-5
-10
-10
-5
0
5
10

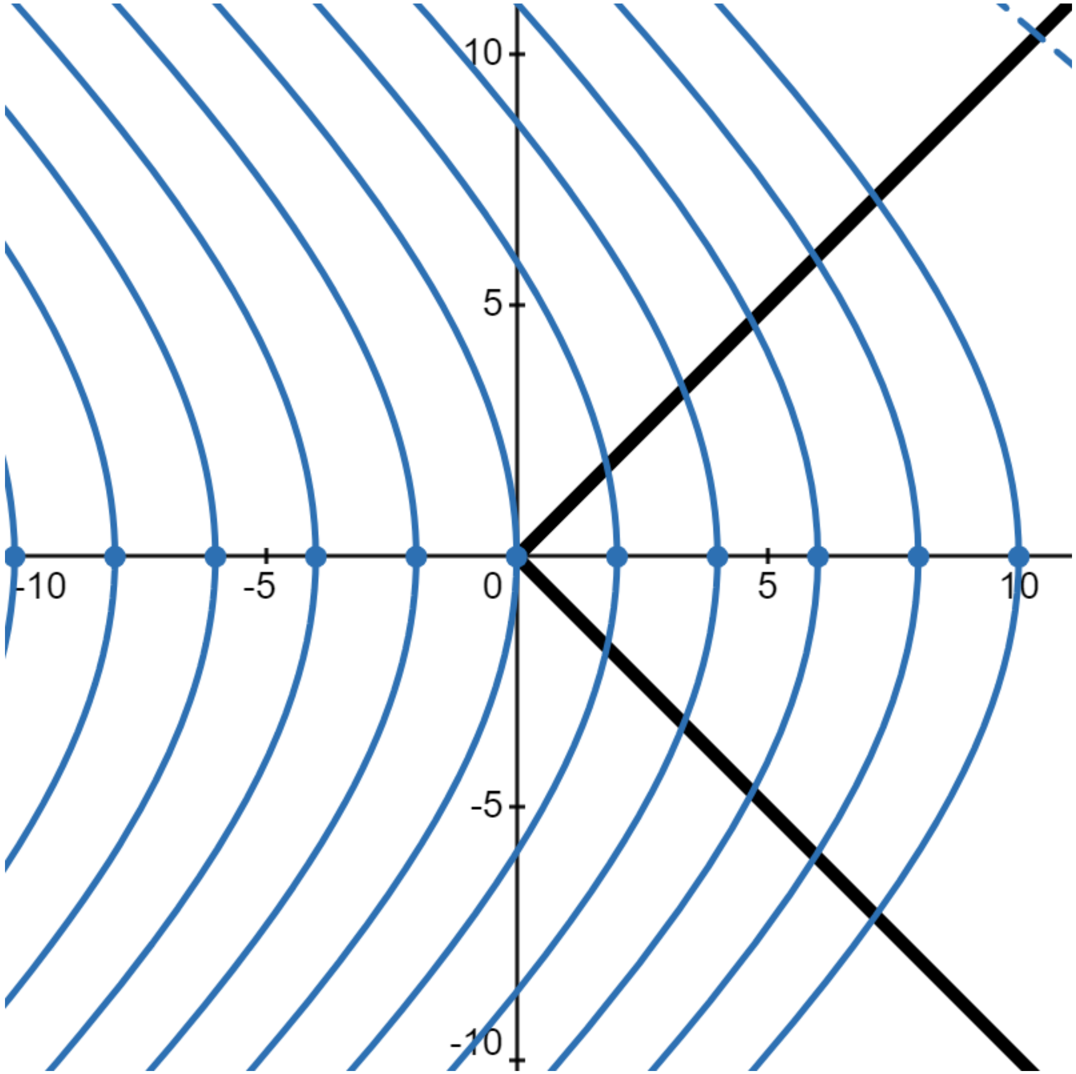

10
5
-5
-10
-10
-5
0
5
10

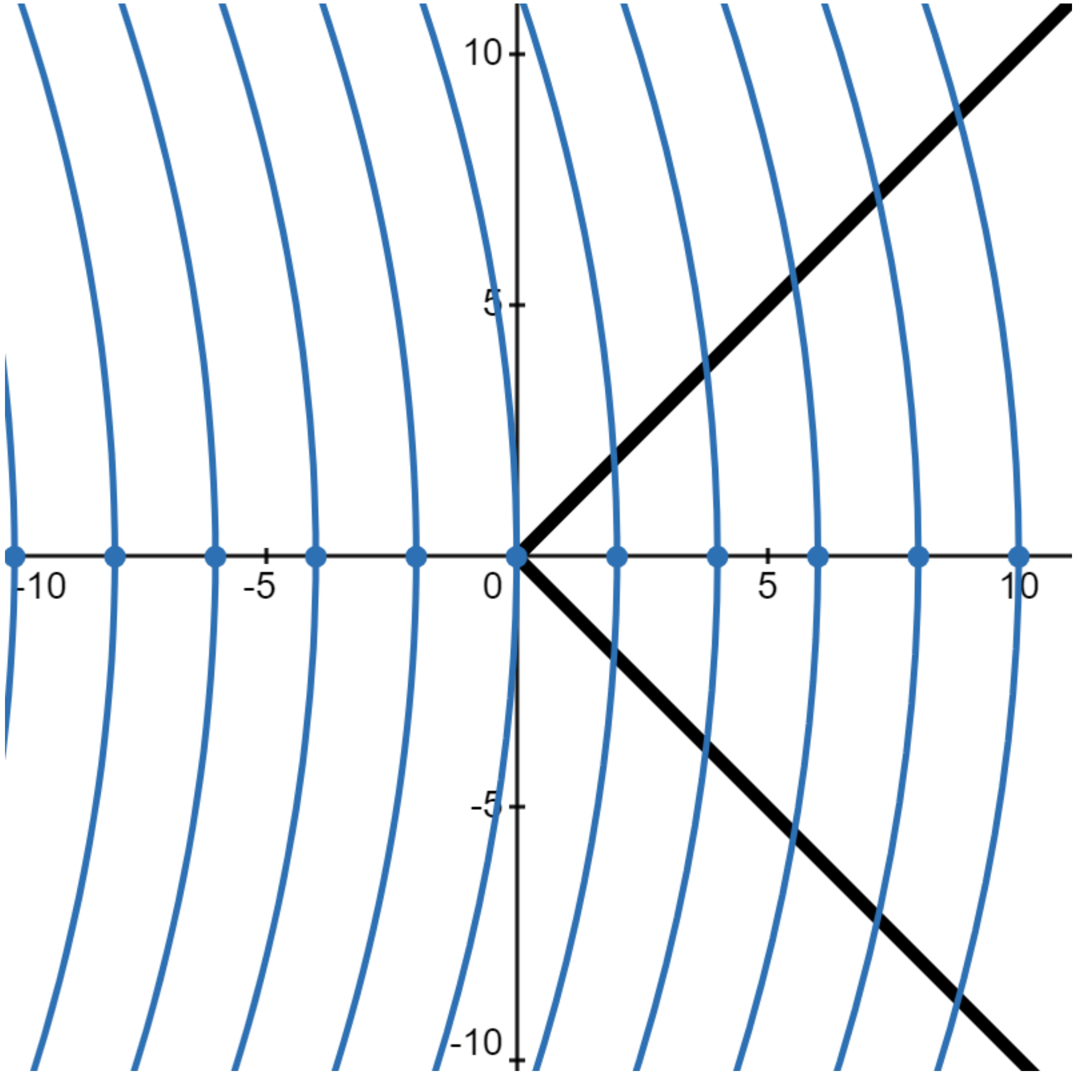


10
5
-5
-10
-10
-5
0
5
10

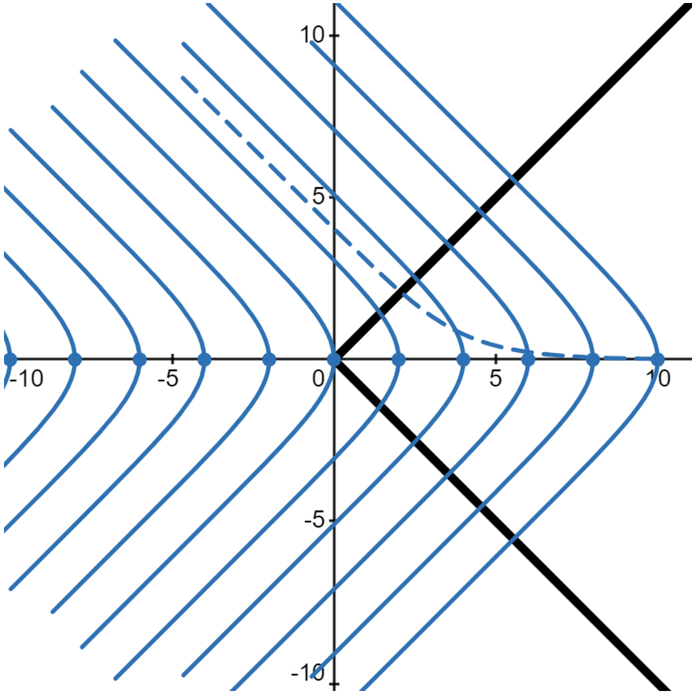


10
5
-5
-10
-10
-5
0
5
10


10
5
-5
-10
-10
-5
0
5
10


10
5
-5
-10
-10
-5
0
5
10

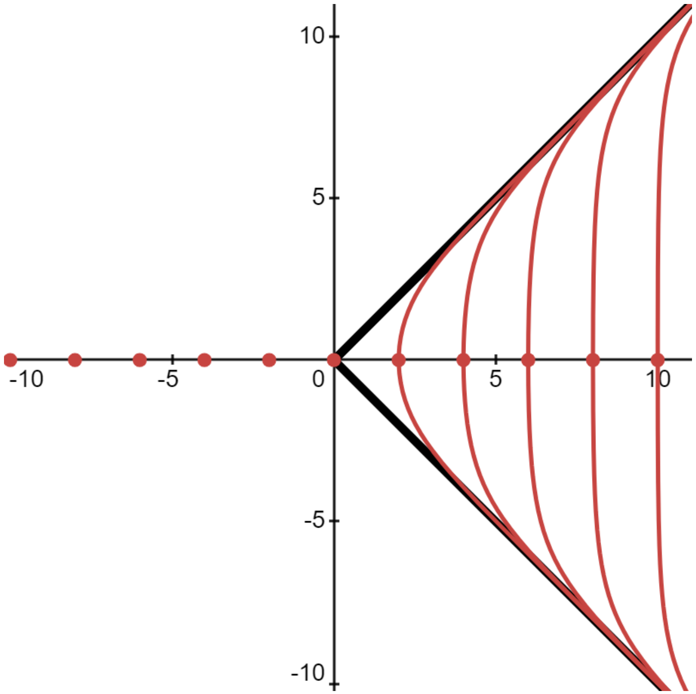
Example: Free Chiral Theory



- An example calculated by Borchers and Yngvason (1999): a 2-dimensional free chiral theory, so $\mathcal{A} = \mathcal{A}_L \otimes \mathcal{A}_R$ where $\mathcal{A}_L$ and $\mathcal{A}_R$ consist of fields existing on the Left and Right light ray.
- If we consider the thermal state at temperature β then the evolution we get is given by

$$\gamma_t^{(R)}(x_R) = \frac{\beta}{2\pi} \ln \left(e^{2\pi x_R/\beta} - t \right)$$
$$\gamma_t^{(L)}(x_L) = -\frac{\beta}{2\pi} \ln \left(e^{-2\pi x_L/\beta} + t \right)$$

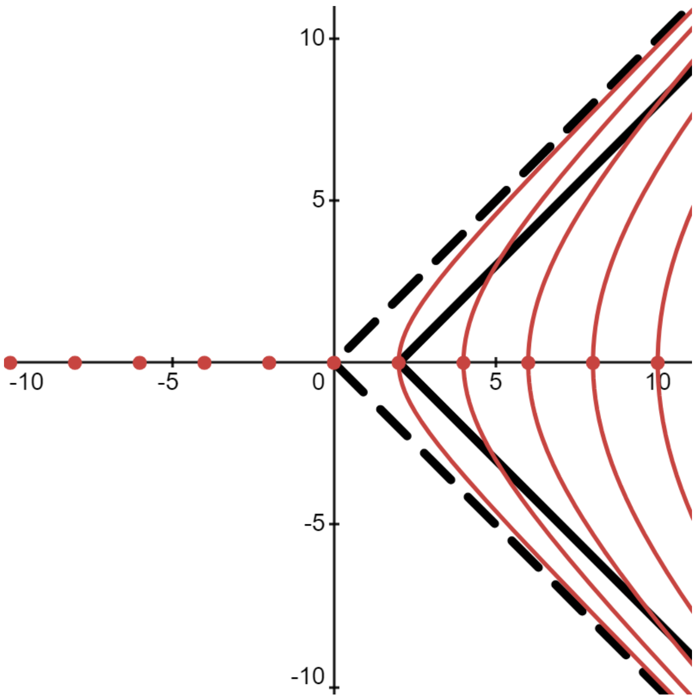
Example: Free Chiral Theory



- An example calculated by Borchers and Yngvason (1999): a 2-dimensional free chiral theory, so $\mathcal{A} = \mathcal{A}_L \otimes \mathcal{A}_R$ where $\mathcal{A}_L$ and $\mathcal{A}_R$ consist of fields existing on the Left and Right light ray.
- If we consider the thermal state at temperature β then the evolution we get is given by

$$\gamma_t^{(R)}(x_R) = \frac{\beta}{2\pi} \ln \left(e^{2\pi x_R/\beta} - t \right)$$
$$\gamma_t^{(L)}(x_L) = -\frac{\beta}{2\pi} \ln \left(e^{-2\pi x_L/\beta} + t \right)$$

Generalization

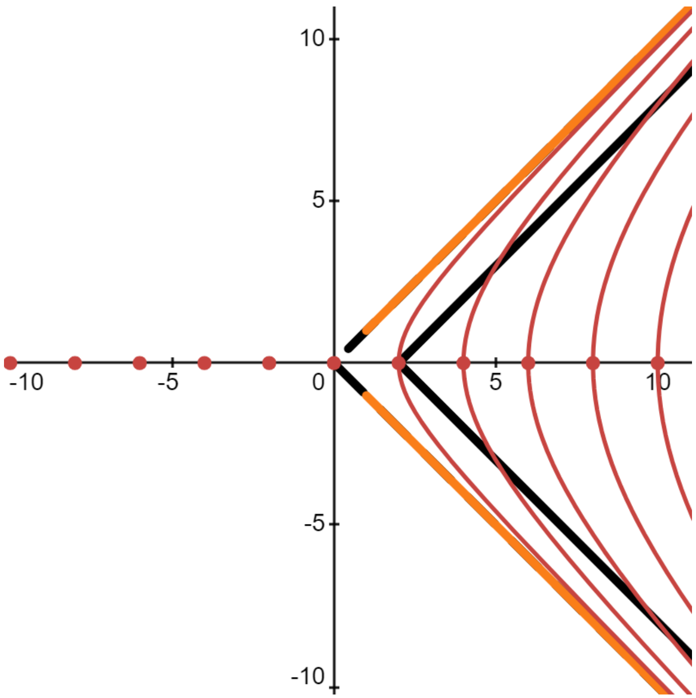


- We can generalize this further to the following situation: let $N_i \subset M_i \subset B(\mathcal{H}_i)$ be Von Neumann algebras with $\Omega_i \in \mathcal{H}_i$ standard for N_i and M_i ($i = 1, 2$). In addition, assume that

$$\sigma_t^{(1)}(N_1) \subset N_1 \text{ and } \sigma_{-t}^{(2)}(N_2) \subset N_2 \text{ for } t \geq 0$$

called Halfsided Modular Inclusion.

Generalization

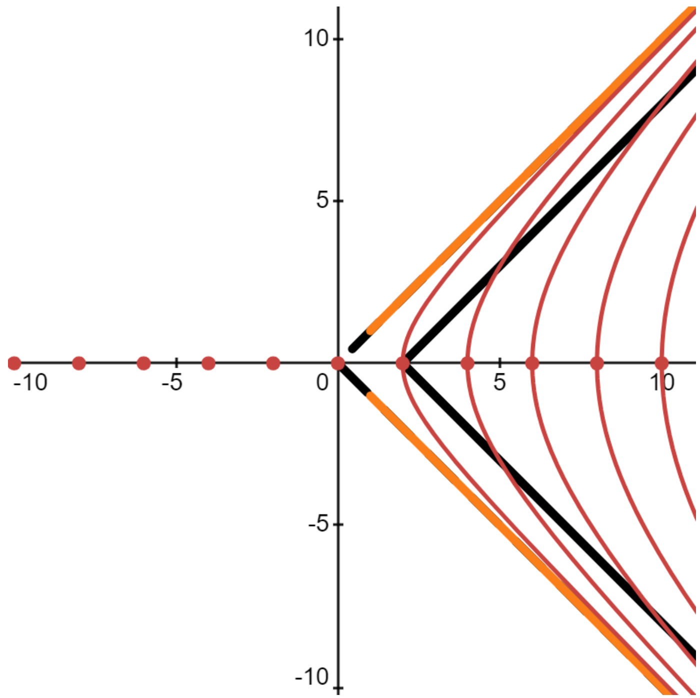


- We can generalize this further to the following situation: let $N_i \subset M_i \subset B(\mathcal{H}_i)$ be Von Neumann algebras with $\Omega_i \in \mathcal{H}_i$ standard for N_i and M_i $(i = 1, 2)$. In addition, assume that

$$\sigma_t^{(1)}(N_1) \subset N_1 \text{ and } \sigma_{-t}^{(2)}(N_2) \subset N_2 \text{ for } t \geq 0$$

called Halfsided Modular Inclusion.

Generalization



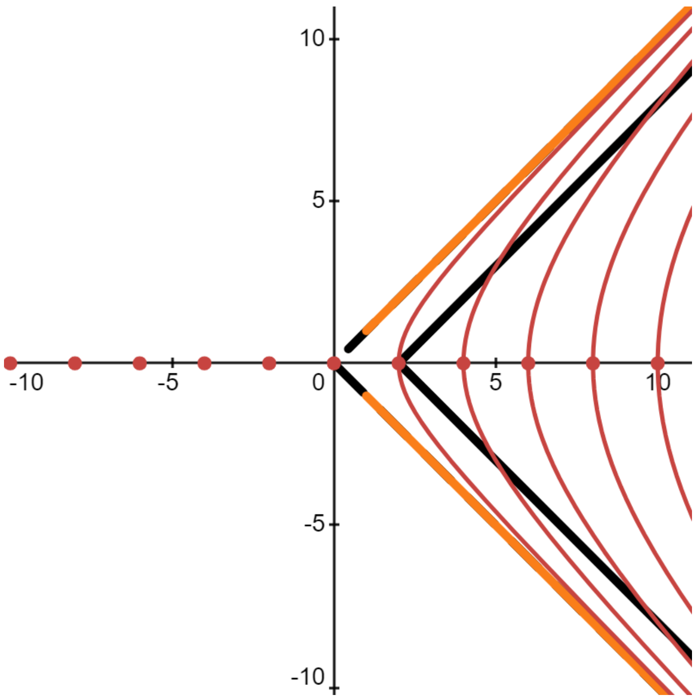
- We can generalize this further to the following situation: let $N_i \subset M_i \subset B(\mathcal{H}_i)$ be Von Neumann algebras with $\Omega_i \in \mathcal{H}_i$ standard for N_i and M_i $(i = 1, 2)$. In addition, assume that

$$\sigma_t^{(1)}(N_1) \subset N_1 \text{ and } \sigma_{-t}^{(2)}(N_2) \subset N_2 \text{ for } t \geq 0$$

  called Halfsided Modular Inclusion.
- In this situation, we have two unitary one-parameter groups U_i on $\mathcal{H}_i$ with a positive generator such that

$$U_1(1)M_1U_1(1)^* = N_1 \quad U_2(-1)M_2U_2(-1)^* = N_2$$

Generalization



- We can generalize this further to the following situation: let $N_i \subset M_i \subset B(\mathcal{H}_i)$ be Von Neumann algebras with $\Omega_i \in \mathcal{H}_i$ standard for N_i and M_i $(i = 1, 2)$. In addition, assume that

$$\sigma_t^{(1)}(N_1) \subset N_1 \text{ and } \sigma_{-t}^{(2)}(N_2) \subset N_2 \text{ for } t \geq 0$$

  called Halfsided Modular Inclusion.
- In this situation, we have two unitary one-parameter groups U_i on $\mathcal{H}_i$ with a positive generator such that

$$U_1(1)M_1U_1(1)^* = N_1 \quad U_2(-1)M_2U_2(-1)^* = N_2$$

- This means that $\vec{x} \mapsto U_1(x_0 - x_1) \otimes U_2(x_0 + x_1)$ is a positive energy representation! Can we generalize this further?

Friedrich-Alexander-Universität
Erlangen-Nürnberg

Converse Unruh effect: Wedge-modular inclusions and thermal states

DPG-Frühjahrstagung Berlin

Ian Koot