# Quantum Non-Causality in Spacetime May Be Not Exclusively Quantum

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This talk is based on the following articles:

- <sup>1</sup> J. Tolksdorf, RV, *Quantum physics, fields and closed timelike curves: The D-CTC condition in quantum field theory* Commun. Math. Phys. **357**, 319-351 (2018)
- <sup>2</sup> J. Tolksdorf, RV, *The D-CTC condition is generically fulfilled in classical (non-quantum) statistical systems* Found. Phys. **51**, 93 (2021)
- <sup>3</sup> A. Much, RV, *Superluminal local operations in quantum field theory: A ping-pong ball test* Universe **9**, 447 (2023)

# **Statistical (Physical) Theories**

Set of random variables  $X \in \mathcal{X}$ 

Describe observations and measurements by correlation functions  $C \in \mathcal{C}$ 

$$
C(X_1, X_2, \ldots, X_n) \in \mathbb{C} \qquad (X_j \in \mathcal{X})
$$

E.g. expectation values, (conditional) probabilities

Typically:

 $\mathcal{X} \leftrightarrow$  set of observables  $\mathcal{C} \leftrightarrow$  set of states

Positive probabilities and multi-linearity in the  $X_i \implies$ 

 $\mathcal{X} \simeq \mathcal{A}$  a \*-algebra

 $\mathcal{C}\simeq \mathcal{S}\subset \mathcal{A}_+^*$  positive linear functionals on $\mathcal{A}$ 

# **Example 1 – classical statistical theories**

 $\mathcal{A}=\pmb{C}^{\pmb{0}}(\mathcal{T})\qquad$  commutative algebra ,  $\qquad \mathcal{T} =$  topological space  $X \leftrightarrow f : \mathcal{T} \rightarrow \mathbb{C}$ 

$$
C(X_1, X_2, ..., X_n) = C(f_1, f_2, ..., f_n) = \langle f_1 \cdot f_2 \cdots f_n \rangle_{\mu}
$$
  
=  $\int_{\mathcal{T}} f_1(\xi_1) f_2(\xi_2) \cdots f_n(\xi_n) d\mu(\xi_1, \xi_2, ..., \xi_n)$ 

 $\mu$  = any probability measure on (the Borel sets of)  $\mathcal T$ 

$$
\langle f \rangle_{\mu} = \int_{\mathcal{T}} f \, d\mu \quad \text{is a state on } \mathcal{A} = C^0(\mathcal{T})
$$
\n
$$
\langle f^* f \rangle_{\mu} \ge 0 \text{ with } f^* = \overline{f}, \qquad \langle 1 \rangle_{\mu} = 1
$$

set of states  $S \leftrightarrow$  set of probability measures on (Borel sets of)  $T$ 

# **Example 2 – quantum statistical theories**

 $\mathcal{A} =$  a non-commutative \*-algebra (or  $C^*$ -algebra), with unit **1** 

$$
X\ \leftrightarrow\ A\in\mathcal{A}
$$

$$
C(X_1, X_2, \ldots, X_n) = C(A_1, A_2, \ldots, A_n) = \langle A_1 \cdot A_2 \cdots A_n \rangle_{\omega}
$$
  
=  $\omega(A_1 \cdot A_2 \cdots A_n)$ 

 $\omega =$  any (sufficiently regular) state on A, where a state is a linear functional  $\omega: A \to \mathbb{C}$  with

 $\omega(A^*A) \ge 0$  and  $\omega(1) = 1$ 

Standard example in quantum physics:

 $A \subset B(H)$ ,  $\omega(A) = \langle A \rangle_{\alpha} = \text{Tr}(\rho A)$ ,  $\rho =$  any density matrix

set of states  $S \leftrightarrow$  set of density matrices on H

# **Some key points:**

• In both classical and quantum case:

The set of states  $S$  is closed under convex combinations and (suitable types of) limits.

A convex combination of states  $\omega_1, \ldots, \omega_n$  in S with weights  $\lambda_1, \ldots, \lambda_n > 0$ is the state

 $\lambda_1\omega_1 + \lambda_2\omega_2 + \ldots + \lambda_n\omega_n \in \mathcal{S} \qquad (\lambda_1 + \ldots + \lambda_n = 1)$ 

The theorems by Gelfand, Naimark, Segal and Wightman establish correspondences

(1) set of all states on  $A \leftrightarrow$  set of all Hilbert space

representations of A for *C* ∗ -algebras

(2)  $A = C^0(\mathcal{T})$  for commutative C<sup>\*</sup>-algebra A

then: set of pure states on  $A \leftrightarrow$  set of points  $\xi \in \mathcal{T}$ 

# 1 — Introduction (1.1)

# **Algebraic /quantum/ field theory on a fixed spacetime manifold**

*M* = a (4-dim) spacetime manifold, e.g. Minkowski spacetime (or any globally hyperbolic spacetime)

*J* <sup>±</sup>(*p*) = set of all *q* ∈ *M* on any future(+)/past(–) directed worldline emanating from *p* ∈ *M*

$$
J^{\pm}(S) = \bigcup_{p \in S} J^{\pm}(p) \text{ for } S \subset M
$$

 $O =$  open interior of  $J^+(p) \cap J^-(p')$  for  $p' \in J^+(p)$  *"double cone"* 



## 1 — Introduction (1.2)

In algebraic quantum field theory (or algebraic *classical* field theory), there is a local structure for the observables:

 $A =$ \*-algebra of (or: generated by) observables,

formed by \*-subalgebras

 $A(O) =$  algebra of observables that can be

measured in the spacetime region *O*

with the properties:

$$
\bullet \ \ O_1 \subset O_2 \quad \Longrightarrow \quad \mathcal{A}(O_1) \subset \mathcal{A}(O_2)
$$

• 
$$
O_2 \cap J^{\pm}(O_1) = \emptyset \implies [A_1, A_2] = 0
$$
 for all  $A_j \in \mathcal{A}(O_j)$ 

• For every symmetry (isometry)  $L : M \to M$  of the spacetime, there is an automorphism  $\alpha_l : A \rightarrow A$  so that

$$
\alpha_L(\mathcal{A}(O)) = \mathcal{A}(L(O)) \quad \text{and} \quad \alpha_{L_1} \circ \alpha_{L_2} = \alpha_{L_1 L_2}
$$

The algebra A may be non-commutative (quantum case) or commutative (classical case)

# **Typical situation in QFT:**

- $\bullet$   $\mathcal{A}(O)$  are weakly closed \*-subalgebras of  $B(\mathcal{H})$  ("von Neumann" algebras")
- **•** Set of (physical) states  $\omega \in S$  given by density matrices  $\rho$  on H:

$$
\omega(A) = \langle A \rangle_{\varrho} = \text{Tr}(\varrho A)
$$

## $\bullet$

 $\alpha_L(\mathcal{A}) = \mathcal{U}_L \mathcal{A} U_L^*$  with continuous unitary group repr  $L \mapsto \mathcal{U}_L$ 

- **•** There is a unit vector  $\psi_0 \in \mathcal{H}$  with  $U_1 \psi_0 = \psi_0$
- **•** static and geodesic time-translations have positive generators: I.e. if  $U_t = e^{itH}$  implements time-shifts of an inertial time-coordinate, then  $H > 0$ .

This is the setting we will adopt in the following, mainly for  $M =$  Minkowski spacetime.

# A very simple quantum circuit



View this as bipartite quantum system with Hilbert spaces  $\mathcal{H}_A$  and  $\mathcal{H}_B$  $\mathcal{H} = \mathcal{H}_{\mathbf{A}} \otimes \mathcal{H}_{\mathbf{B}}$  $U: \mathcal{H} \rightarrow \mathcal{H}$  unitary

−*T* symbolizes "step backward in time", meaning that partial state of full system after applying *U* is the same as before applying *U* on system part *B*

Given a unitary U on H and a partial state (density matrix)  $\rho_A$  on system part *A*, a state (density matrix)  $\rho$  of the full system is said to **fulfill the D-CTC condition** if the restriction of  $\rho$  to system part *A* coincides with  $\rho_A$  and if  $U_{\mathcal{Q}}U^*$  and  $\varrho$  agree when restricted to system part *B*.

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Given: *U* unitary on H,  $\rho_A$  density matrix on  $\mathcal{H}_A$ 

A density matrix  $\rho$  on  $H$  **fulfills the D-CTC condition** if

$$
\bullet \quad \mathrm{Tr}_\mathcal{B}\varrho=\varrho_\mathcal{A} \ \Leftrightarrow \ \mathrm{Tr}(\varrho(\bm{a}\otimes \bm{1}))=\mathrm{Tr}_{\mathcal{H}_\mathcal{A}}(\varrho_\mathcal{A}\bm{a})
$$

 $\mathrm{Tr}_A U_Q U^* = \mathrm{Tr}_{A} \varrho \ \ \Leftrightarrow \ \ \mathrm{Tr}(\varrho (\mathbf{1} \otimes \mathbf{b})) = \mathrm{Tr}( U_Q U^* (\mathbf{1} \otimes \mathbf{b}))$ 

David Deutsch has shown: If H*<sup>A</sup>* and H*<sup>B</sup>* are *finite dimensional*, then for any given  $U$  and  $\varrho_A$  there is a  $\varrho$  fulfilling the D-CTC condition.

His argument rests on compactness of the state space = set of density matrices for finite-dimensional Hilbert spaces. This permits to employ a fixed-point argument.

Some (including David Deutsch) have claimed that D-CTC provides a form (or analog) of a **time travel scenario** —

*"...quantum mechanics therefore allows for causality violation without paradoxes whilst remaining consistent with relativity"*

Ringbauer et al., Nature Communications **5** (2014) 4145

...it has also recently gained popularity in pop culture...



Tony Stark – aka Iron Man

dixit:

"Quantum fluctuation messes with the Planck scale, which then triggers the **Deutsch Proposition**"

## **Questions**:

- Is the D-CTC condition characteristic for quantum processes involving CTCs?
- Or is it merely an analogy of certain aspects of CTCs?
- **•** Can the claim by Ringbauer et al. be substantiated or refuted?

The original version of the D-CTC condition makes no reference to spacetime structure (deliberately). To check on the previous questions, translate the setting into algebraic quantum field theory on Minkowski spacetime *M* with its built-in local and causal structure of the local observable algebras A(*O*).

In algebraic QFT:

Bipartite systems are represented by operator algebras  $A(O_A)$  and  $A(O_B)$  for causally separated spacetime regions *O<sup>A</sup>* and *O<sup>B</sup>*



**D-CTC Problem**: Given a unitary U on H and a density matrix state

$$
\omega_A(\mathbf{a}) = \text{Tr}(\varrho_A \mathbf{a}) \quad \text{ on } A(O_A),
$$

is there a density matrix state  $\omega(\mathbf{c}) = \text{Tr}(\rho \mathbf{c})$  on  $B(\mathcal{H})$  whose partial state on  $A(O_A)$  agrees with  $\omega_A$  and which is U-invariant in restriction to  $A(O_B)$ , i.e.

 $\omega(\mathbf{a}) = \omega_A(\mathbf{a})$  on  $\mathcal{A}(O_A)$  and  $\omega(U^*\mathbf{b}U) = \omega(\mathbf{b})$  on  $\mathcal{A}(O_B)$ ) ?

# **Theorem 1 (JT & RV, CMP 357)**

Assume that the QFT fulfills the split property ( $\Leftrightarrow$  density matrix states  $\omega_A$  and  $\omega_B$  are always restrictions of a density matrix state on H without correlations across  $A(O_A)$  and  $A(O_B)$ ).

Then, given any unitary U on H and any density matrix state  $\omega_A(\mathbf{a}) = \text{Tr}(\rho_A \mathbf{a})$ on A(*OA*), there is an *approximate* solution to the D-CTC problem in the following sense:

Given arbitrary  $R > 0$  (large) and  $\epsilon > 0$  (small), there is a density matrix state  $\omega = \omega_{R,\epsilon}$  on  $B(\mathcal{H})$  such that

$$
\bullet\ \omega(\mathbf{a})=\omega_{A}(\mathbf{a})\quad(\mathbf{a}\in\mathcal{A}(O_{A}))
$$

$$
\bullet\ |\omega(U^*{\bf b} U)-\omega({\bf b})|<\epsilon\quad ({\bf b}\in\mathcal{A}(O_B)\,,\ ||{\bf b}||< R)
$$

This indicates that the D-CTC condition is **not** characteristic for occurrence of CTCs since it can be fulfilled to arbitrary precision in QFT on Minkowski spacetime. The proof rests on convexity and approximate completeness (relates to insisting on density matrix states) of the state space in QFT.

# **Operations**

Given: Statistical theory, with observable algebra  $A$ , set of states  $S$ An **operation** is a convex (and weak\*-continuous) map  $\tau : \mathcal{S} \to \mathcal{S}$ Typical example: If  $U \in A$  is **unitary**, then

$$
\tau_U:\omega\mapsto\omega_U\,,\qquad \omega_U(\textbf{a})=\omega(U^*\textbf{a}U)
$$

is an operation (unitary operation).

- Definition of operation applies both for non-commutative or commutative A
- If A is commutative, then unitary operations are trivial:  $\omega_U = \omega$  for every unitary  $U \in \mathcal{A}$ .
- Concept of operation defined here is *non-selective*, or *probability preserving*. Could generalize to selective operations. That would include measurements.

Formulating the **D-CTC condition** for classical statistical systems, with the following given data:

(i) 
$$
\mathcal{T} = \mathcal{T}_A \times \mathcal{T}_B
$$
 with locally compact Hausdorff spaces  $\mathcal{T}_A$ ,  $\mathcal{T}_B$ ;  $\mathcal{A} = C_b(\mathcal{T})$ ,  $\mathcal{A}_A = C_b(\mathcal{T}_A)$ ,  $\mathcal{A}_B = C_b(\mathcal{T}_B)$ 

(ii) An operation  $\tau : \mathcal{S} \to \mathcal{S}$  ( $\mathcal{S} = \text{states}(\mathcal{A})$ )

(iii)  $\omega_A \in S_A$  ( $S_A$  = states( $\mathcal{A}_A$ ))

We say that the **D-CTC condition can be fulfilled** in the system if for any given  $\tau$  and for any given  $\omega_A \in S_A$  there is some  $\omega \in S$  so that

$$
\omega(f_A \otimes 1_B) = \omega_A(f_A) \quad (f_A \in \mathcal{A}_A = C_b(\mathcal{T}_A))
$$
  

$$
\tau(\omega)(1_A \otimes f_B) = \omega(1_A \otimes f_B) \quad (f_B \in \mathcal{A}_B = C_b(\mathcal{T}_B))
$$

Also want:  $\omega$  is a probability measure if  $\omega_A$  is prob. measure and if  $\tau$  maps prob. measures to prob. measures (always fulfilled if  $\mathcal T$  is compact).

# **Theorem 2 (JT & RV, Found. Phys. 51)**

Assumptions:

- $\star$   $\tau_A$  and  $\tau_B$  are locally compact **metric spaces**,
- $\star$   $\tau$  maps probability measures to probability measures,
- $\star \qquad \omega_A = \mu_A$  is a tight probability measure,
- $\star$  there is a probability measure  $\mu_B^\circ$  on  $\mathcal{T}_B$  so that

 $\tau^{n}(\mu_{A} \times \mu_{B}^{\circ}), \quad n \in \mathbb{N}, \quad \text{is tight}$ 

 $(\mu_A \times \mu_B^{\circ} \text{ is the product measure})$ 

Then the D-CTC condition can be fulfilled for the given  $\omega_A$  and  $\tau$  and the state  $\omega$  fulfilling it is given by a Borel probability measure  $\mu$ .

A sequence of Borel probability measures  $\{\mu_n\}_{n\in\mathbb{N}}$  is called **tight** if:

For any  $\varepsilon > 0$  there is a compact set  $\mathcal{K} \subset \mathcal{T}$  so that

 $\mu_n(\mathcal{T}\backslash\mathcal{K})\leq \varepsilon \quad (n\in\mathbb{N})$ 

For a QFT with local observable algebras A(*O*):

If  $O_A$  and  $O_B$  are causally separated  $(O_B \cap J^\pm(O_A) = \emptyset)$  then any unitary operation  $\tau_U$  with  $U \in \mathcal{A}(O_A)$  has no effect on  $\mathcal{A}(O_B)$ :

$$
\omega_U(\mathbf{b}) = \omega(U^* \mathbf{B} U) = \omega(U^* U \mathbf{b}) = \omega(\mathbf{b}) \qquad (\mathbf{b} \in \mathcal{A}(\mathcal{O}_B))
$$

Therefore, such  $\tau_U$  is called a **local operation**, *localized in*  $O_A$ *.* 

Can all such local operations be physically performed?

# 3 — Impossible measurements/operations in QFT (3.2)

If they could – for any unitaries in the local observable algebras – that may lead to **superluminal signalling** (a violation of causality) as pointed out by Raphael Sorkin (1993):

Consider 3 spacetime regions, named after experimenters carrying out measurements/operations therein:



Since *O*Alice and *O*Charlie are causally separated, Charlie cannot know by measuring in *O*Charlie if Alice has carried out a unitary operation  $\tau_{U_{\text{Alice}}}$  with  $U_{\text{Alice}} \in \mathcal{A}(O_{\text{Alice}})$ :

$$
\tau_{U_{\mathrm{Alice}}}(\omega)(\bm{c}) = \omega(U_{\mathrm{Alice}}^*\bm{c}U_{\mathrm{Alice}}) = \omega(\bm{c}) \quad \text{for all $\bm{c} \in \mathcal{A}(O_{\mathrm{Charlie}})$, $\hspace{0.1cm} \omega \in \mathcal{S}$}
$$

But if first Alice carries out a unitary operation, and then Bob, we have:

$$
\tau_{U_{\text{Bob}}} \circ \tau_{U_{\text{Alice}}}(\omega)(\mathbf{c}) = \omega(U_{\text{Alice}}^* U_{\text{Bob}}^* \mathbf{c} U_{\text{Bob}} U_{\text{Alice}}) \text{ for all } \mathbf{c} \in \mathcal{A}(O_{\text{Charlie}})
$$
  
In general,  $U_{\text{Bob}} \in \mathcal{A}(O_{\text{Bob}})$  won't commute with all  $\mathbf{c} \in \mathcal{A}(O_{\text{Charlie}})$   
nor with all  $U_{\text{Alice}} \in \mathcal{A}(O_{\text{Alice}})$  since

 $O_{\text{Alice}}$  causally overlaps with  $O_{\text{Bob}}$  and  $O_{\text{Bob}}$  causally overlaps with  $O_{\text{Charlie}}$ 

Hence, one can choose  $U_{\text{Alice}}$ ,  $U_{\text{Bob}}$ , **c** and  $\omega$  such that

$$
\tau_{U_{\mathrm{Bob}}}\circ \tau_{U_{\mathrm{Alice}}}(\omega)(\boldsymbol{c})\neq \tau_{U_{\mathrm{Bob}}}(\omega)(\boldsymbol{c})
$$

This means, Charlie can determine by measuring the observable *c* in *O*Charlie if Alice has carried out an operation  $\tau_{U_{\text{Alice}}}$  in  $O_{\text{Alice}}$ , if Bob carries out a suitable operation  $\tau_{U_{\text{Bob}}}$  in  $O_{\text{Bob}}$ .

This would mean a superluminal transfer of information since O<sub>Alice</sub> and *O*Charlie are causally separated.

Examples are given in: R. Sorkin (1993); L. Bosten, I. Jubb, G. Kells, PRD 104 (2021); I. Jubb, PRD 105 (2022).

The issue is that  $\tau_{U_{\text{Bob}}}$  amounts to a superluminal communication channel between  $O_{\text{Alice}}$  and  $O_{\text{Charlic}}$  which is unphysical.

But such superluminal communication channels arise also in classical field theory, e.g. by local, kinematical symmetries.

# **Theorem 3 (AM & RV, Universe (2023))**

Let A(*O*) be the local observable algebras of the classical or the quantized Klein-Gordon field on Minkowski spacetime *M*, with field equation  $(\Box + m^2)\varphi = 0.$ 

Then there are states  $\omega$  and operations  $\tau_{\text{Alice}}$  and  $\tau_{\text{Bob}}$  together with observables  $\boldsymbol{c} \in \mathcal{A}(O_{\text{Charlie}})$  so that

$$
\tau_{\mathrm{Bob}} \circ \tau_{\mathrm{Alice}}(\omega)(\bm{c}) \neq \tau_{\mathrm{Bob}}(\omega)(\bm{c})
$$

 $\tau_{\text{Alice}}$  and  $\tau_{\text{Bob}}$  are localized in  $O_{\text{Alice}}$  and  $O_{\text{Bob}}$ , i.e.  $\tau_{\text{Bob}}(\tilde{\omega})$  (*d*) =  $\tilde{\omega}(\textbf{d})$  if  $d \in A(O_d)$  with  $O_d$  causally separated from  $O_{B_0b}$ .

Specifically,  $\tau_{\text{Bob}}$  can be chosen so that it corresponds to an instantaneous rotation around the *x* 3 -axis by 180 degrees, flipping *O*(−) ↔ *O*(+) (local kinematical symmetry).

For the quantized Klein-Gordon field, there is a unitary  $U_{\text{Bob}} \in \mathcal{A}(O_{\text{Bob}})$  so that

$$
\tau_{\rm Bob}(\omega)(\,.\,) = \omega(\textit{U}^\ast_{\rm Bob}\,.\,\textit{U}_{\rm Bob})
$$



 $\tau_{\text{Bob}}$  has the effect of flipping  $O^{(-)}$  instantaneously to  $O^{(+)}$  and vice versa.

## **Remarks**

**•** The approach of describing classical field theory in terms of a local algebra framework has been developed by Brunetti, Duetsch, Fredenhagen and Rejzner (and co-authors). See:

K. Rejzner: *Perturbative Algebraic Quantum Field Theory*, Springer, 2016

M. Duetsch: *From Classical Field Theory to Perturbative Quantum Field Theory*, Birkhäuser, 2019

**In the** *classical* case,  $\tau_{\text{Bob}}$  and  $\tau_{\text{Alice}}$  are not implemented by unitaries in the local algebras since the local algebras are commutative — they are formed by (certain) functions on the phase space.

The generator of  $\tau_{\text{Bob}}$  can be obtained with the help of a *Peierls bracket*, generalising the Poission bracket of Hamiltonian mechanics.

- Original setting for D-CTC condition does not refer to spacetime structure; does not relate to CTCs in the sense of GR
- Thm 1 shows that D-CTC is not characteristic for occurrence of CTCs in the sense of GR.
- Thm 2 shows that **D-CTC can always be fulfilled in classical (non-quantum) statistical systems** – it is more a generalized ergodic theorem than related to quantum mechanics.

Put bluntly: **The D-CTC condition has nothing to with quantum mechanics** (uncertainty relations, interference, entanglement) but only relates to the basic statistical setting of quantum mechanics.

The *impossible measurements/impossible operations scenario* does not only arise in QFT, but also in classical field theory.

There are "superluminal" local operations also in classical field theory, e.g. by local kinematical symmetries. Not all local operations in quantum or classical field theory can be "actively" carried out.

# 4 — Conclusion (4.2)

We have carried out the **ping-pong ball test**<sup>∗</sup> on "D-CTC" and "impossible operations/measurements" (and it failed in both cases) —

*When someone presents a paradox as being rooted in quantum physics,*

*replace the term quantum mechanical particle by ping-pong ball everywhere.*

*If the paradox persists, it is unrelated to quantum physics.*

But this does not mean that "D-CTC" and "impossible operations/measurements" are not interesting. They point to issues that need to be better understood in QFT.

The "impossible measurements scenario" can be avoided in more recent approaches towards QFT measurements:

C.J. Fewster, RV, Comm. Math. Phys. 378 (2020);

H. Bostelmann, C.J. Fewster, M. Ruep, PRD 103 (2021);

M. Papageorgiou, D. Fraser, Found. Phys. 53 (2024)

<sup>∗</sup> Due to Reinhard Werner (oral version)