Interplay between boundary conditions and the Lorentzian Wetterich equation

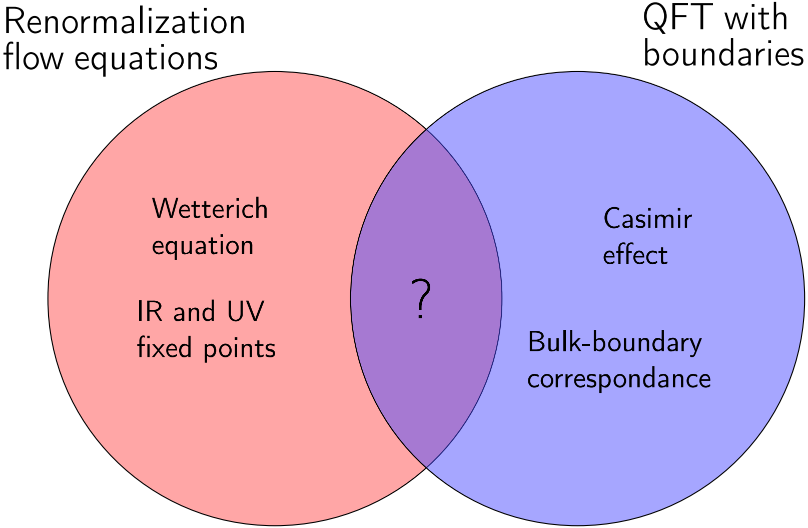
Filippo Nava

University of York

Based on: C. Dappiaggi, F.N., L. Sinibaldi
(Rev. in Math. Phys., 2024)

LQP 49, Erlangen 2024

Context of the work



Renormalization flow and boundaries

Aim of the work

Studying the renormalization flow fixed points of the scalar quantum field, with quartic self-interaction, on a spacetime with boundary.

$$I(\chi) = -\int_{\mathcal{M}} d\mu_g \, \nabla_a\chi\nabla^a\chi + \frac{m^2}{2}\chi^2 + \frac{\xi}{2}R\chi^2 + \frac{\lambda}{4!}\chi^4$$

Done employing a novel renormalization flow equation (Lorentzian Wetterich equation[a])

[a] E. D'Angelo, N. Drago, N. Pinamonti, K. Rejzner (Annales Henri Poincaré, 2024)

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Spacetimes with boundary

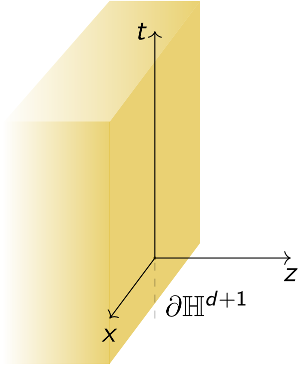
Lorentzian manifolds with a time-like boundary

$\mathcal{M}$ manifold with boundary, $(\mathcal{M}, g)$ Lorentzian manifold.

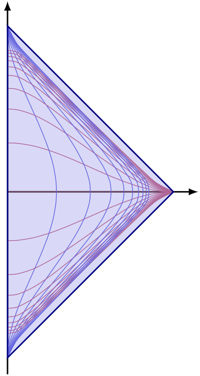
$(\mathcal{M}, g)$ is a Lorentzian manifold with time-like boundary if $(\partial\mathcal{M}, i^*g)$ is a Lorentzian manifold, where $i : \partial\mathcal{M} \hookrightarrow \mathcal{M}$.

Examples

Half Minkowski $(\mathbb{H}^{d+1}, \eta)$

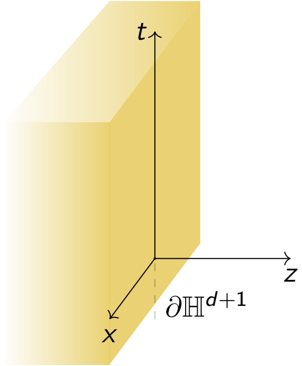


Anti de-Sitter $(\mathrm{P}\mathbb{A}\mathrm{d}\mathbb{S}_{d+1}, g)$



Half Minkowski spacetime

Half Minkowski $(\mathbb{H}^{d+1}, \eta)$



$$ds^2_{\mathbb{H}^{d+1}} = -dt^2 + dz^2 + \sum_{i=1}^{d-1} x_i^2$$

$$z \geq 0,\ x_i \in \mathbb{R}.$$

Poincaré patch of Anti de-Sitter

$$-X_0^2 - X_{d+1}^2 + \sum_{i=1}^{d} X_i^2 = -l^2,$$

$$l^2 = -\frac{d(d+1)}{\Lambda}$$

$$\Downarrow$$

$$ds^2_{\mathrm{P}\mathbb{A}\mathrm{d}\mathbb{S}} = \frac{l^2}{z^2}(-dt^2 + dz^2 + \sum_{i=1}^{d-1} x_i^2)$$

$$z > 0,\ x_i \in \mathbb{R}.$$

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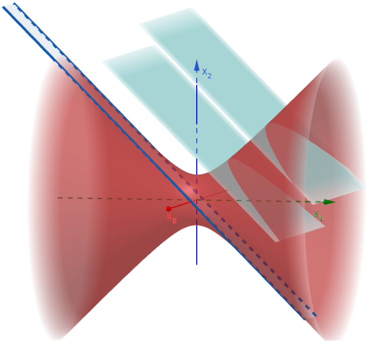

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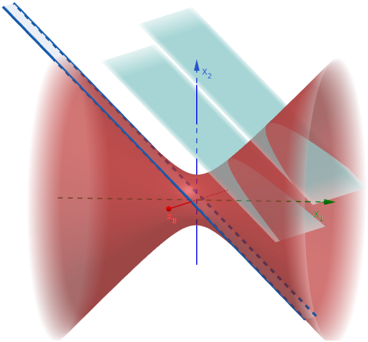

Conformal relation

$$ds^2_{\mathrm{P}\mathbb{A}\mathrm{d}\mathbb{S}} = \frac{l^2}{z^2}(-dt^2+dz^2+\sum_{i=1}^{d-1} x_i^2)$$

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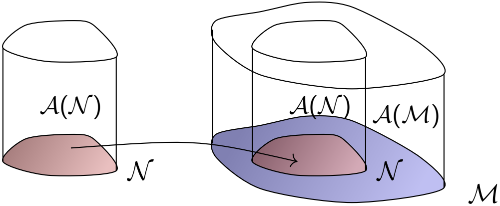
Algebraic QFT in the functional approach

$(\mathcal{M}, g)$ Lorentzian manifold $\rightarrow \mathcal{A}(\mathcal{M})$ algebra of observables



Functional approach

$C^\infty(\mathcal{M})$ classical field configurations

$\mathcal{A}(\mathcal{M}) \subset \mathcal{F}(\mathcal{M})$ functionals on $C^\infty(\mathcal{M})$, $\quad F : C^\infty(\mathcal{M}) \rightarrow \mathbb{R}$

$\mathcal{F}_{loc}(\mathcal{M})$ local functionals:

$$X_f^n(\chi) = \int_{\mathcal{M}} \chi^n f, \qquad f \in C_c^\infty(\mathcal{M}), \;\; \chi \in C^\infty(\mathcal{M})$$

Deformation quantization

Classical product: $F \cdot G \quad \xrightarrow{\hbar} \quad$ Quantum product: $F \star G$

1. (Associativity) $\star$ is an associative product;
2. (Classical limit) $F \star G \xrightarrow{\hbar \to 0} F \cdot G$;
3. (Weak Dirac prescription) $\{F, G\} \longrightarrow \frac{1}{i\hbar}[F, G]_\star + \mathcal{O}(\hbar^2)$.

Algebra structure on $\mathcal{F}_{loc}(\mathcal{M})$

Klein-Gordon operator $P = \Box_g - m^2 - \xi R$. If $\mathcal{M}$ is *globally hyperbolic*:

- $\exists \; \Delta_+ \in \mathcal{D}'(\mathcal{M} \times \mathcal{M})$ weak bisolution of KG
- $\exists \; H \in \mathcal{D}'(\mathcal{M} \times \mathcal{M})$ with the same singular structure of Δ_+

Unique up to the choice of a smooth function: physical state

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Algebra structure on $\mathcal{F}_{loc}(\mathcal{M})$

Quantum structure on $\mathcal{F}_{loc}(\mathcal{M})$

Given $F, G \in \mathcal{F}_{loc}(\mathcal{M})$ and Δ_+ a 2-point function we define

$$F \star G = F \cdot G + \sum_{n=1}^{\infty} \frac{\hbar^n}{n!} \langle F^{(n)}, \Delta_+^{\otimes n} G^{(n)} \rangle$$

Normal ordering of $X_f^2 \equiv \chi^2(x)$

$$:\chi^2(x):= \lim_{x \to y} \chi(x) \star \chi(y) - \hbar H(x, y)$$

as $\Delta_+ - H \in \ C^\infty(\mathcal{M})$

Interacting theory

Perturbative AQFT

Interacting action functional $I_0(\chi) + V(\chi)$,

$$V(\chi) = \lambda \int_{\mathcal{M}} \chi^4 f,\ \lambda \in \mathbb{R}$$

Thanks to Epstein-Glaser renormalization we can define the time-ordered product $\cdot_T$ and the local S-matrix

$$S(V) \doteq \sum_{n=0}^{\infty} \frac{i^n}{\hbar^n n!} [\underbrace{:V: \ \cdot_T \ldots \cdot_T \ :V:}_{n \text{ times}}]$$

AQFT on spacetimes with a boundary

The initial-value problem must be supplemented with boundary conditions:

$\chi = 0$ on $\partial\mathcal{M}$ (Dirichlet) $\quad$ $\nabla_n\chi = 0$ on $\partial\mathcal{M}$ (Neumann)

Quantum structure on $\mathcal{F}_{loc}(\mathcal{M})$

On $(\mathbb{H}^4, \eta)$ and $(\mathrm{P}\mathbb{A}\mathrm{d}\mathbb{S}_4, g)$ with Dirichlet/Neumann BCs it holds that [a] [b]:

- $\exists\ \Delta_+$ 2-point function
- $\exists\ H$ Hadamard function

[a] O. Gannot, M. Wrochna (Journal of the Institute of Mathematics of Jussieu, 2022)
[b] C. Dappiaggi, A. Marta (Mathematische Nachrichten, 2022)

Lorentzian Wetterich equation

In the pAQFT formalism it holds the Lorentzian Wetterich equation[1]

$$\partial_k \Gamma_k(\phi) = -\lim_{x \to y} k \int_{\mathcal{M}} d\mu_g(y)\, \mathrm{Sym}\, \Delta_{+,k}(x,y) - H_k(x,y)$$

$$\partial_k \Gamma_k^{(2)}|_{\phi=0} = -\int_{\mathcal{M}} d\mu_g \partial_k m_k^2$$

$$\partial_k \Gamma_k^{(4)}|_{\phi=0} = -\int_{\mathcal{M}} d\mu_g \partial_k \lambda_k$$

Fixed point: $\partial_k m_k^2 = \partial_k \lambda_k = 0$

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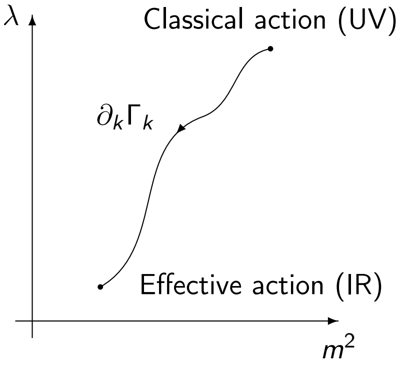

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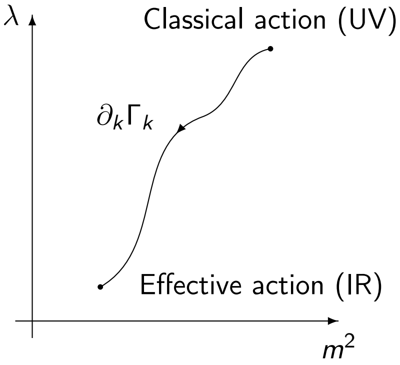

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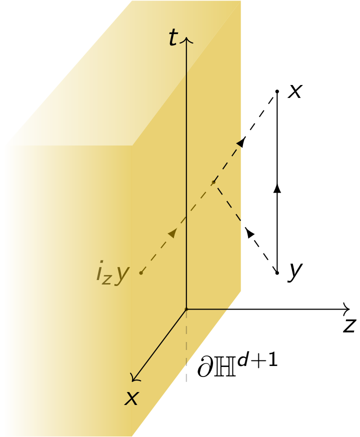

With Dirichlet/Neumann BCs:

$$\Delta_+(x,y) = \Delta_+^{\mathbb{M}}(x,y) \mp \Delta_+^{\mathbb{M}}(x, i_z y)$$

1. $H(x,y) = H^{\mathbb{M}}(x,y) \mp \underbrace{H^{\mathbb{M}}(x, i_z y)}_{\text{smooth at } x=y}$

   Full subtraction

2. $H(x,y) = H^{\mathbb{M}}(x,y)$

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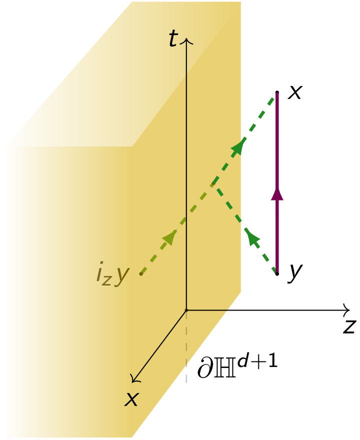

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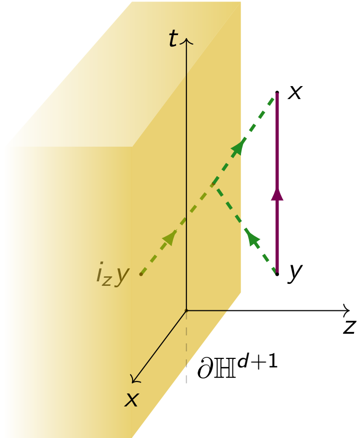

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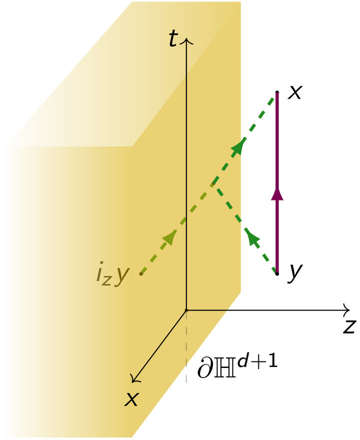

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Scaling of full Minkowski

$$
\begin{cases}
k\partial_k \widetilde{m}_k^2 = -2\widetilde{m}_k^2 + \dfrac{\widetilde{\lambda}_k}{16\pi^2}\Big[\left(\log\left(\widetilde{m}_k^2+1\right)+1\right)\left(1\mp\mathfrak{B}\left(\widetilde{z}\right)\right) \\
\qquad \pm\left(1+\widetilde{m}_k^2\right)\log\left(1+\widetilde{m}_k^2\right)\mathfrak{B}_2\left(\widetilde{z}\right)\Big] \\
k\partial_k \widetilde{\lambda}_k = \dfrac{\widetilde{\lambda}_k^2}{16\pi^2}\Big[\dfrac{3}{1+\widetilde{m}_k^2}\left(1\mp\mathfrak{B}\left(\widetilde{z}\right)\right) \\
\qquad \mp 6\left(\log\left(\widetilde{m}_k^2+1\right)+1\right)\mathfrak{B}_2\left(\widetilde{z}\right) \\
\qquad \pm\left(1+\widetilde{m}_k^2\right)\log\left(\widetilde{m}_k^2+1\right)\mathfrak{B}_4\left(\widetilde{z}\right)\Big]
\end{cases}
$$

where $\mathfrak{B}(\widetilde{z}) \doteq \frac{\sqrt{2}}{iMz} I_1(\sqrt{2}iMz)$, $\widetilde{m}_k^2 = k^{-2}m_k^2$, $\widetilde{\lambda}_k = \lambda_k$, $\widetilde{z} = kz$

Full subtraction - Dirichlet/Neumann BCs

$$
\begin{cases}
k\partial_k \widetilde{m}_k^2 = -2\widetilde{m}_k^2 + \dfrac{\widetilde{\lambda}_k}{16\pi^2}\Big[\left(\log\left(\widetilde{m}_k^2+1\right)+1\right)\ \left(1 \mp \mathfrak{B}\left(\widetilde{z}\right)\right) \\
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Minimal subtraction - Dirichlet/Neumann BCs

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where $\mathfrak{C}(\widetilde{z}) \doteq \frac{2}{Mz}K_1(2zM)$, $\widetilde{m}_k^2 = k^{-2}m_k^2$, $\widetilde{\lambda}_k = \lambda_k$, $\widetilde{z} = kz$

Comparison between full and minimal subtraction

$$\mathfrak{B}(\tilde{z}) \in \mathbb{R}$$

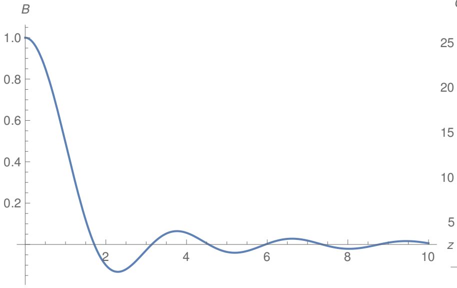
$$\lim_{\tilde{z}\to\infty} \mathfrak{B}(\tilde{z}) = 0$$

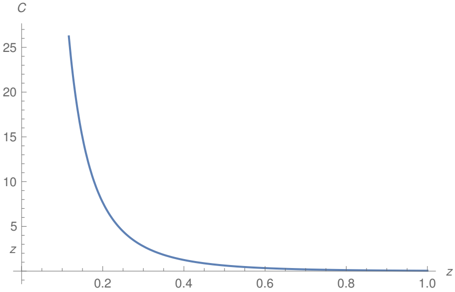
$$|\mathfrak{B}(\tilde{z})| \leq 1$$

$$\mathfrak{C}(\tilde{z}) \in \mathbb{R}$$

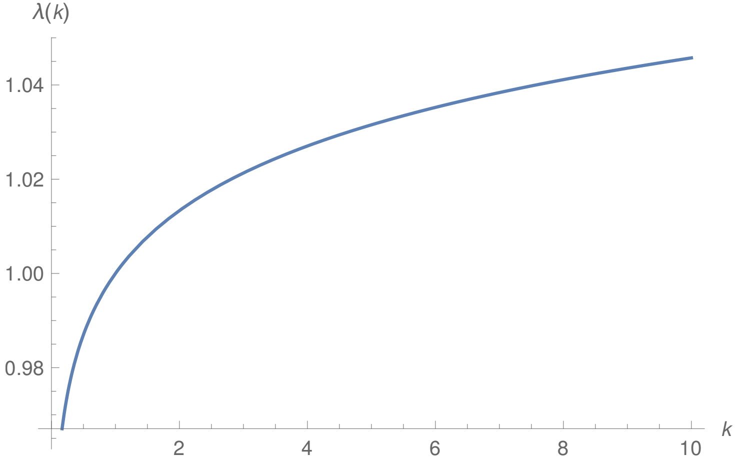
$$\lim_{\tilde{z}\to\infty} \mathfrak{C}(\tilde{z}) = 0$$

$$\mathfrak{C}(\tilde{z}) \xrightarrow{\tilde{z}\to 0} +\infty$$

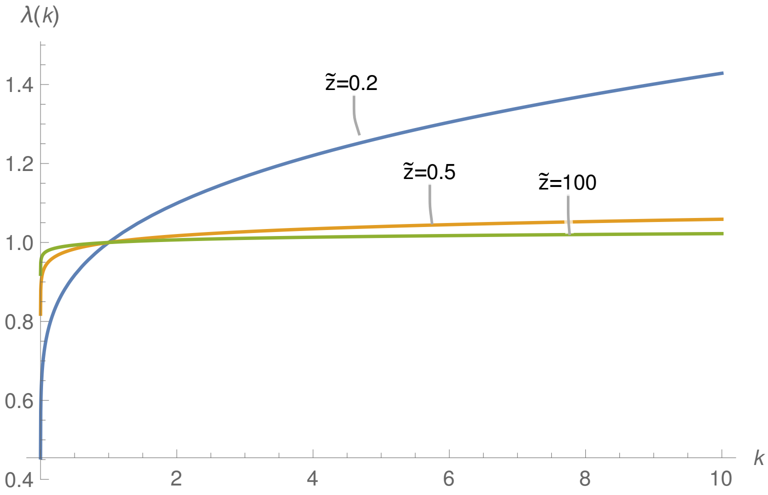




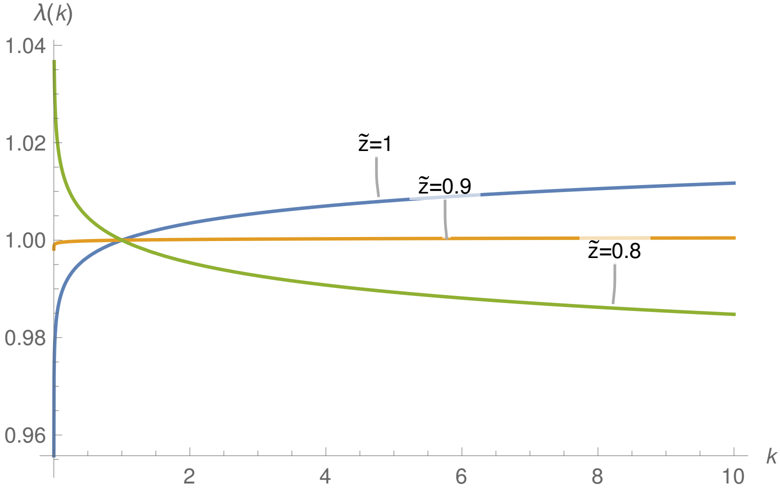
Scaling of λ on full Minkowski



Minimal subtraction - Dirichlet BCs



Minimal subtraction - Neumann BCs



Poincaré patch of Anti de-Sitter

To construct $\Delta_+,\ H$:

$$\Omega^2 g_{\mathrm{P}\mathbb{A}\mathrm{d}\mathbb{S}} = \eta_{\mathbb{H}}, \quad \Omega = \frac{z}{l}$$

If $\chi : \mathrm{P}\mathbb{A}\mathrm{d}\mathbb{S}_{d+1} \to \mathbb{R}$ satisfies $(\Box_g - m^2 - \xi R)\chi = 0$

$$\Downarrow$$

$$\overline{\chi} = \Omega^{\frac{1-d}{2}}\chi : \mathbb{H}^{d+1} \to \mathbb{R} \text{ satisfies } \left(\Box_\eta - \frac{\overline{m}^2}{z^2}\right)\overline{\chi} = 0, \ \overline{m}^2 = m^2 + \left(\xi - \tfrac{d-1}{4d}\right)R$$

Flow on Anti de-Sitter - Dirichlet BCs

$$\begin{cases} k\partial_k \widetilde{m}_k^2 = \dfrac{\widetilde{\lambda}_k}{4\tilde{\nu}}\left[1 - \left(\tilde{\nu}^2 - \dfrac{1}{4}\right)(\dots)\right] \\ k\partial_k \widetilde{\lambda}_k = 2\widetilde{\lambda}_k + \dfrac{3\widetilde{\lambda}_k^2}{16\tilde{\nu}^3}\left[-2 - \left(\tilde{\nu}^2 - \dfrac{1}{4}\right)(\dots)\right] \end{cases}$$

where $(\dots) \propto \psi, \psi', \psi''$ digamma function, $\widetilde{m}_k^2 = l^2 m_k^2$, $\widetilde{\lambda}_k = k^2 l^2 \lambda_k$, $\tilde{\nu} = \frac{1}{2}\sqrt{1 + 4k^2 l^2 + 4\widetilde{m}_k^2}$

Large Λ approximation

If $\Lambda \to -\infty$, as $l^2 \propto -\frac{1}{\Lambda}$: $k^2 l^2 = 0 \to$ autonomous ODE

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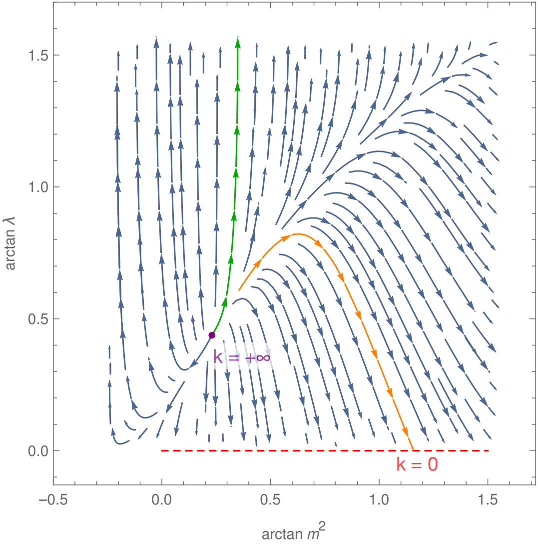
Flow on Anti de-Sitter - Dirichlet BCs

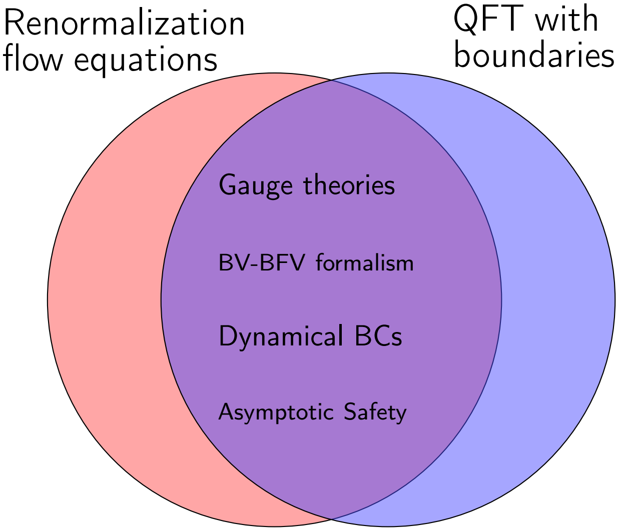


Summary

Results

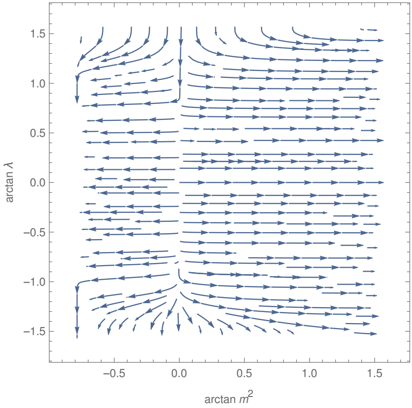
- State-dependence of UV/IR behaviour
- Asymptotic freedom "near the boundary" with Neumann BCs
- Set of non-interacting, exact IR fixed points on $\mathrm{P}\mathbb{A}\mathrm{d}\mathbb{S}_4$
- UV fixed point under the large Λ approximation

Future prospectives

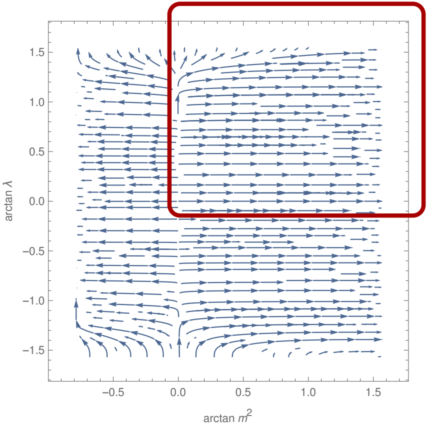


Comparison between Minkowski and the full subtraction

Minkowski

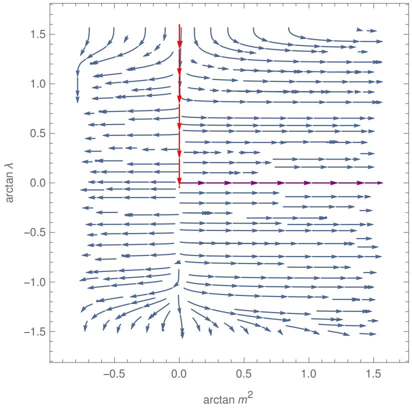


Half Minkowski, full subtraction:
$\widetilde{z} = 10^{-3}$

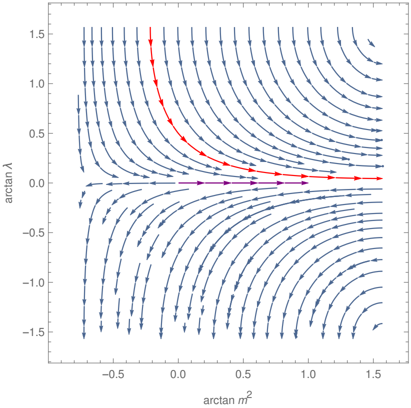


Comparison between Minkowski and the minimal subtraction (Dirichlet BCs)

Minkowski

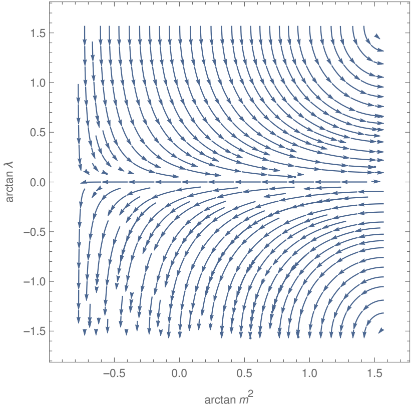


Half Minkowski: $\widetilde{z} = 10^{-5}$

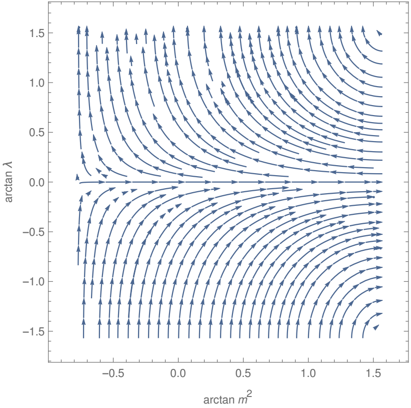


Comparison between Dirichlet and Neumann BCs

Dirichlet



Neumann



Hadamard parametrix

Theorem (Radzikowski)

For x, y in a *convex normal neighborhood* $\mathcal{C} \subset \mathcal{M}$ there exist $U, V, W \in C^\infty(\mathcal{M} \times \mathcal{M})$ such that

$$H(x,y) = \underbrace{\frac{U(x,y)}{\sigma(x,y)} + V(x,y)\log\left(\frac{\sigma(x,y)}{\mu^2}\right)}_{\text{fixed by the geometry}} + \underbrace{W(x,y)}_{\text{physical state}}$$

Hadamard function

Given

- *convex normal neighborhood* $C \subset M$;
- σ_ϵ *geodesic distance*;
- Hadamard parametrix

$$H_\epsilon(x,y) = \frac{u(x,y)}{(2\pi)^2 \sigma_\epsilon(x,y)} + v(x,y) \log\left(\frac{\sigma_\epsilon(x,y)}{\lambda^2}\right);$$

there exists $s \in C^\infty(C \times C)$ such that for all $f, g \in C^\infty(C)$:

$$\lim_{\epsilon \to 0^+} \int_{C \times C} H_\epsilon(x,y) Pf(x) g(y) dxdy = \int_{C \times C} s(x,y) f(x) g(y) dxdy$$